Indian Astronomy

An Introduction

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This book is dedicated to the two great pioneers of Indian Astronomy,

Āryabhaṭa I and Brahmagupta,

on this significant occasion of the 15th centenary of the ĀRYABHAṬĪYAM and the 14th birth centenary of Brahmagupta.

DIACRITICAL MARKS FOR ROMAN TRANSLITERATION OF DEVANAGARI

Short Vowels					*Long Vowels*						*Visarga*	
अ	इ	उ	ऋ	लृ (ळ)	आ	ई	ऊ	ए	ओ	ऐ	औ	:
a	*i*	*u*	*ṛ*	*ḷ*	*ā*	*ī*	*ū*	*e*	*o*	*ai*	*au*	*ḥ*

Consonants

क्	ख्	ग्	घ्	ङ्	च्	छ्	ज्	झ्	ञ्
k	*kh*	*g*	*gh*	*ṅ*	*c*	*ch*	*j*	*jh*	*ñ*
ट्	ठ्	ड्	ढ्	ण्	त्	थ्	द्	ध्	न्
ṭ	*ṭh*	*ḍ*	*ḍh*	*ṇ*	*t*	*th*	*d*	*dh*	*n*
प्	फ्	ब्	भ्	म्					
p	*ph*	*b*	*bh*	*m*					

Compound Consonants

क्ष्	त्र्	ज्ञ्	य्	र्	ल्	व्	श्	ष्	स्	ह्
kṣ	*tr*	*jñ*	*y*	*r*	*l*	*v*	*ś*	*ṣ*	*s*	*h*

Contents

Preface

The present book on Indian Astronomy is designed mainly for the benefit of students and general readers. The purpose of the book is not only to create an awareness about Indian astronomy among the readers, but also to make them fairly proficient in the concepts, technicalities and computational procedures developed by great Indian mathematicians and astronomers over the past two thousand years.

Most of the concepts used in Indian astronomy like *sauramāna*, *candramāna*, lunar calendars, *tithis*, *nakṣatras*, *saṅkrānti*, *yugādi*, etc., are encountered by every Indian in his day-to-day life. A discerning person would be interested in knowing more about these concepts and many such issues of astronomical interest that form an important part of Indian culture. There is a great dearth of suitable books which systematically explain important and interesting topics of Indian astronomy to students in a simple and intelligible fashion. No doubt, important classical Sanskrit texts like the **Sūryasiddhānta** and **Siddhāntaśiromaṇi** are critically edited and even translated into English, Hindi and some other Indian languages. But these texts are generally beyond the grasp of a modern reader.

It is strongly suggested and hoped that the contents of the present book will be made a part of the curriculum at the school and college levels. This thoughtful decision on the part of our educationists and textbook writers would provide the much-needed opportunity for our students to get a reasonably good grasp of the important aspects of Indian astronomy.

Organisation and Content

The development of astronomy in India – from the Vedic times to the recent times – is briefly surveyed in Chapter 1.

The basic concepts of the celestial sphere, diurnal motion, etc., are discussed in Chapter 2. The different co-ordinate systems, viz., celestial longitude and latitude, right ascension and declination, azimuth and altitude, the phenomenon and effects of precession of equinoxes constitute Chapter 3.

The zodiac, *rāśi* and *nakṣatra* systems are explained in Chapter 4. Division of time into convenient units like civil day and sidereal day, solar and civil calendars, solar and lunar months, *adhikamāsa* and *ksayamāsa* (intercalary months), six *ṛtus* (seasons), *yogas* and time on a microcosmic scale are explained in Chapter 5.

Calendar systems, viz., the Gregorian, Hindu and Muslim solar and lunar systems and the five limbs (*pañcāṅga*) – *tithi, vāra, nakšatra, yoga* and *karaṇa* – form the subject matter of Chapter 6.

The method of finding *ahargaṇa* (number of civil days from a chosen epoch), number of revolutions of heavenly bodies in a *mahāyuga* (of 432×10^4), computations of mean longitude of the Sun, the Moon and the five planets (Mars, Mercury, Jupiter, Venus and Saturn) and the *deśāntara* correction (due to difference in terrestrial longitudes) are explained in Chapter 7.

From the mean positions of the Sun and the Moon, the true positions can be obtained by applying the 'equation of centre' (*mandaphala*). For this purpose, Indian astronomers have prescribed the peripheries of the *manda* epicycles of different heavenly bodies. These aspects as well as the true daily motions of the Sun and the Moon, the *Bhujāntara* correction, etc., are elaborated in Chapter 8.

In the case of the five planets, in addition to the *mandaphala* (equation of centre), one more important equation called *śīghra samskāra* (equation of 'conjunction') has to be applied. Here again, Indian astronomers have considered the peripheries of *śīghra* epicycles. Chapter 9 deals with these equations for the determination of true longitudes of the planets as well as their true daily motions, retrograde motion and the *Bhujāntara* correction for the so-called *tārāgrahas* (star-planets).

Topics related to the three important issues (*tripraśna*), viz., direction, place and time (*dik, deśa* and *kāla*) are examined in Chapter 10. These include the determination of the latitude of a place, sunrise and sunset times, rising of the signs of the zodiac and finding the *Lagna* (ascendant) at a given time and place.

The cause of lunar eclipses, the angular diameter of the Earth's shadow-cone, the ecliptic limits for lunar eclipse and half-durations of a lunar eclipse and of maximum obscuration, form the subject matter of Chapter 11.

Chapter 12 deals with the cause of solar eclipse, the angular distance between the Sun and the Moon at the beginning and end of a solar eclipse, computation of solar eclipse according to **Sūryasiddānta**, *saros* and metonic cycles.

A unique and pioneering feature of the present book is providing ready-to-use ***computer programs*** to compute: (i) true positions of the Sun, the Moon and the Moon's node (*Rāhu*), (ii) lunar eclipse, (iii) solar eclipse and (iv) true positions of five planets. For these computations, the algorithms as given in the popular **Sūryasiddhānta** are adopted.

A fairly exhaustive ***Bibliography*** and ***Index*** are included at the end of the book to enhance the usefulness of the text and also to motivate the interested readers to go deeper into the subject.

In addition, *Glossaries* of technical words from English to Sanskrit and vice versa are also provided for ready reference.

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I acknowledge my indebtness to all the authors and publishers of the works listed in the Bibliography. Valuable suggestions are indeed welcome from discerning readers.

1

Historical Introduction

"Like the crests on the heads of peacocks, like the gems on the hoods of the cobras, *Jyotiṣa* (Astronomy) is at the top of the **Vedāṅga sastras**—the auxiliary branches of Vedic knowledge".

Vedāṅgajyotiṣa, 4

1.1 Introduction

The above verse shows the singular importance given to astronomy (and mathematics) over the other branches of knowledge in the Vedic times.

The night sky filled with innumerable bright stars and planets, the "wanderers", has been the object of constant curiosity and excitement to man ever since the beginning of civilization. During the day, the rising Sun in the east and the setting Sun in the west, so also the periodically waxing and waning Moon at nights, drew the attention of the observer in man.

All these recurring phenomena as well as the annual repetitions of the seasons acquainted the early astronomers with their periodicity. In the course of time, further interesting phenomena like the solar and lunar eclipses, the visits of comets, etc., intrigued the astronomers to an even greater extent. And they began formulating the laws that govern these phenomena. These early developments of astronomy seem to have taken place more or less simultaneously in the different ancient civilizations—the Egyptian, the Mesopotamian and the Indian.

1.2 Ancient Indian Astronomy

Like many other branches of knowledge, the origins of the science of astronomy in India have to be traced back to the *Vedas*. In the Vedic literature, *Jyotiṣa* is one of the six

auxiliaries (*shadangas*) of the Vedic corpus of knowledge. The six *vedāṅgas* are:

(*i*) *Śikṣa* (phonetics)
(*ii*) *Vyākaraṇa* (grammar)
(*iii*) *Chandas* (metrics)
(*iv*) *Nirukta* (etymology)
(*v*) *Jyotiṣa* (astronomy) and
(*vi*) *Kalpa* (rituals).

It is important to note that although in modern common parlance the word *Jyotiṣa* is used to mean predictive astrology, in the earlier literature *Jyotiṣa* included all aspects of astronomy. Of course, mathematics was regarded as a part of *Jyotiṣa*. **Vedāṅgajyotiṣa** is the earliest Indian astronomical text available.

1.3 THE VEDIC PERIOD AND VEDĀṄGAJYOTIṢA

The **Vedāṅgajyotiṣa** was mainly used to fix suitable times for performing different kinds of sacrifices. The text is found in two rescensions—**Ṛgveda** *Jyotiṣa* and **Yajurveda** *Jyotiṣa*. Though the contents of both the rescensions are the same, they differ in the number of verses. While the Ṛgvedic version contains only 36 verses, the Yajurvedic version contains 44 verses. This difference in the number of verses is perhaps due to the addition of explanatory verses by the *Adhvaryu* priests by whom it was used.

In one of the verses, it says, "I shall write on the lore of time, as enunciated by sage Lagadha". Therefore, the authorship of **Vedāṅgajyotiṣa** is attributed to Lagadha.

According to the text, at the time of its composition, the winter solstice was at the beginning of the constellation *Śrāviṣthā* (Delphini) and the summer solstice was in the middle of the *Āśleṣā* constellation. Since Varāhamihira (505 AD) stated that in his own time the summer solstice was at the end of three quarters of *Punarvasu* and the winter solstice at the end of the first quarter of *Uttarāṣāḍhā*, there had been a precession of the equinoxes (and solstices) by one and three-quarters

of a *nakṣatra*, i.e., about 23°20′. Since the rate of precession is about a degree in 72 years, the time interval for a precession of 23°20′ is about 72 × 23°20′; i.e., 1680 years prior to Varāhamihira's time. This takes us back to around 1150 BC. The generally agreed upon period of the **Vedāṅgajyotiṣa** is between the 12th and 14th centuries BC.

The **Vedāṅgajyotiṣa** belongs to the last part of the Vedic age. The text can be considered as a record of the fundamentals of astronomy necessary for the day-to-day life of the people of those times. The **Vedāṅgajyotiṣa** is the culmination of the knowledge developed and accumulated over thousands of years of the Vedic period upto 1400 BC.

Even as early as the time of the *maṇḍalas* of the **Ṛgveda**, the Vedic people were conversant with the knowledge required for their religious activities, like the times (and periodicity) of the full and the new moons, the last disappearance of the Moon and its first appearance, etc. This type of information was necessary for monthly rites like *darśapūrṇamāsa* and seasonal rites like *cāturmāsya*.

The *nakṣatra* system consisting of 27 *nakṣatras* (or 28 including *Abhijit*) was evolved long ago and was used to indicate days. It is pointed out that *Agrahāyaṇa*, an old name for the *Mṛgaśira nakṣatra*, meaning "beginning of the year" suggests that the Sun used to be in that asterism at the time of the vernal equinox. This corresponds to the period around 4000 BC.

The *Rohiṇī* legends in the **Ṛgveda** point to a time in the late Ṛgvedic period when the vernal equinox shifted to the *Rohiṇī* asterism (from *Mṛgaśira*).

The sacrificial ritual called *Gavamayaṇa* was especially designed for the daily observation of the movements of the Sun and of the disappearance of the Moon. This must have given the priests and their advisors knowledge of a special kind, rather like the "saros" of the Greeks, for predicting the eclipses. There is evidence, in the **Ṛgveda**, that specialized knowledge about the eclipses was possessed by the priests of the *Atri* family.

During the **Yajurveda** period, it was known that the solar year has 365 days and a fraction more. In the **Taittirīya Samhitā**, it is mentioned that the extra 11 days over the twelve lunar months (totalling 354 days), complete the six *ṛtus* by the performance of the *ekadasa-ratra*, i.e., eleven-nights sacrifice. Again, the same **Samhitā** says that 5 days more

were required over and above the *sāyana* year of 360 days to complete the seasons, adding specifically that: "4 days are too short and 6 days too long".

The Vedic astronomers evolved a system of five years' *yuga*. The names of the five years of a *yuga* are:

(*i*) *Samvatsara*
(*ii*) *Parivatsara*
(*iii*) *Idāvatsara*
(*iv*) *Anuvatsara*, and
(*v*) *Idvatsara*.

This period of a *yuga* (of 5 years) was used to calculate time as can be seen from statements like, "Dīrghatamas, son of Mamatā, became old even in his tenth *yuga*", i.e., between the age of 45 and 50 years (**Ṛgveda**, 1.158.6).

In the **Yajurveda**, a year comprising 12 solar months and 6 *ṛtus* (seasons) was recognized. The grouping of the six *ṛtus* and the twelve months, in the Vedic nomenclature, is as follows:

	Seasons	**Months**
(1)	*Vasanta ṛtu*	*Madhu* and *mādhava*
(2)	*Grīṣma ṛtu*	*Śukra* and *Śuci*
(3)	*Varṣa ṛtu*	*Nabhas* and *Nabhasya*
(4)	*Śarad ṛtu*	*Isha* and *Urja*
(5)	*Hemanta ṛtu*	*Saha* and *Sahasya*
(6)	*Śiśira ṛtu*	*Tapa* and *Tapasya*

The sacrificial year commenced with *Vasanta ṛtu*. The Vedic astronomers had also noted that the shortest day was at the winter solstice when the seasonal year *Śiśira* began with *Uttarāyaṇa*, and rose to the maximum at the summer solstice.

In the **Vedāṅgajyotiṣa** a *yuga* of 5 solar years consists of 67 lunar sidereal cycles, 1830 days, 1835 sidereal days, 62 synodic months, 1860 *tithis*, 135 solar *nakṣatras*, 1809 lunar *nakṣatras* and 1768 risings of the Moon. It also mentions that there are 10 *ayanas* and *viṣuvas* and 30 *ṛtus* in a *yuga*.

The practical way of measuring time is mentioned as the time taken by a specified quantity of water to flow through the opening of a

specified clepsydra (water-clock), as one *nāḍikā*, i.e., $^1/_{60}{}^{th}$ part of a day. The **Vedāṅgajyotiṣa** also has a useful classification of those times like:

(*i*) the solstices
(*ii*) increase and decrease of the durations of days and nights in the *ayanas*
(*iii*) the solstitial *tithis*
(*iv*) the seasons
(*v*) omission of *tithis*
(*vi*) table of *parvas*
(*vii*) *yogas* (which developed later as one of the five limbs of a full-fledged *pañcāṅga*)
(*viii*) finding the *parva nakṣatras* and the *parva tithis*
(*ix*) the *viṣuvas* (equinoxes)
(*x*) the solar and other types of years
(*xi*) the revolutions of the Sun and the Moon (as seen from the Earth)
(*xii*) the times of the Sun's and the Moon's transit through a *nakṣatra*
(*xiii*) the *adhikamāsa* (intercalary month)
(*xiv*) the measures of the longest day and the shortest night, etc.

The **Vedāṅgajyotiṣa** mentions that the durations of the longest and the shortest days on the two solstices are 36 and 24 *ghaṭikā* (*nāḍikās*) which correspond to 14 hours, 24 minutes and 9 hours, 36 minutes, respectively. This means the *dinārdha*, i.e. the length of half-day, comes to be 7h 12m and 4h 48m, respectively, which differs from 6h by 1h 12m. This is called the ascensional difference. It is calculated that around 1400 BC, the Sun's maximum declination used to be about $23°53'$. However, our ancient Indian astronomers took it as $24°$. Now, the latitude, ϕ, of a place can be found using the formula:

$$\text{Sin (ascensional difference)} = \tan\phi \tan\delta$$

where δ is the declination of the Sun. The correction due to the ascensional difference in this case is 1h 12m, i.e. in angular measure, $1\text{h } 12\text{m} \times 15° = 18°$. Now using the above formula, we get the latitude of the place as $\phi = 35°$, approximately. Therefore, the place of composition of the **Vedāṅgajyotiṣa** appears to be in some region around the northern latitude of $35°$.

1.4 *SIDDHĀNTA*

The astronomical computations described in the **Vedāṅgajyotiṣa** were in practical use for a very long time. Around the beginning of the Christian era, say a century on either side of it, a new type of Indian astronomical literature emerged. The texts representing this development are called *siddhānta*. The word "*siddhānta*" has the connotation of an established theory. These *siddhānta* texts contain much more material and deal with more topics than the **Vedāṅgajyotiṣa**.

Along with the *nakṣatra* system, the twelve signs of the zodiac, viz. *Meṣa, Vṛṣabha,* etc. were introduced. A more precise value for the length of the solar year was adopted. Computations of the motions of the planets, the solar and lunar eclipses, ideas of parallax, determination of mean and true positions of planets and a few more topics formed the common contents of the siddhāntic texts.

A very significant aspect of that period, and of the history of Indian astronomy, was the remarkable development of newer mathematical methods which greatly promoted mathematical astronomy. Needless to say, the unique advantage of the famous Hindu decimal numbers made even computations with huge numbers very simple, and even enjoyable, to the ancient Indian astronomers.

According to the Indian tradition, there were principally 18 *siddhāntas*: *Sūrya, Paitāmaha, Vyāsa, Vāsiṣṭha, Atri, Parāśara, Kāśyapa, Nārada, Gārgya, Marīchi, Manu, Āṅgīra, Lomaśa* (or *Romaka*), *Pauliśa, Cyavana, Yavana, Bhrigu* and *Śaunaka*. However, among these, only five *siddhāntas* were extant during the time of Varāhamihira (505 AD) viz., *Saura* (or *Sūrya*), *Paitāmaha* (or *Brahma*), *Vāsiṣṭha, Romaka* and *Pauliśa*. These five *siddhāntas* were ably compiled by Varāhamihira and preserved for posterity as his **Pañcasiddhāntikā**.

1.5 ĀRYABHAṬA I (476 AD)

Āryabhaṭa I, to be distinguished from his namesake of the tenth century, was born in 476 AD and composed his very famous work, **Āryabhaṭiyam**, when he was just 23 years old. He

mentions in this monumental text that he sets down the knowledge honoured at Kusumapura, identified with modern Patna in Bihar.

The **Āryabhaṭiyam** consists of four parts (*pādas*): *Gītikā, Gaṇita, Kālakriya* and *Gola.* The first part contains 13 verses and the remaining three parts, forming the main body of the text, contain altogether 108 verses.

In the *Gitikāpāda*, we are introduced to (*i*) the large units of time, viz. *Kalpa, Manvantara* and *Yuga* (different from those of the **Vedāṅga-jyotiṣa**), (*ii*) circular units of arc – degrees and minutes and (*iii*) linear units, viz., *Yojana, Hasta* and *Aṅgula.*

The numbers of revolutions of planets in a *mahāyuga* of 43,20,000 years are given in the *Gītikāpāda.* Further, the positions of the planets, their apogees (or aphelia) and nodes are also given. Besides these, the diameters of the planets, the inclinations of the orbital planes of the planets with the ecliptic, and the peripheries of the epicycles of the different planets are also included. The topic of mathematical importance, in this part, is the construction of the tables of *Jyā*, the trigonometric function 'Sine'. It is significant that so much information is packed, as if in a concentrated capsule form, into just ten verses.

The second part of the **Āryabhaṭiyam**, the *Gaṇitapāda* contains 33 stanzas essentially dealing with mathematics. This part deals with the following important mathematical topics: geometrical figures, their properties and mensuration (*Kṣetra vyavahāra*); arithmetic and geometric progressions; problems on the shadow of the gnomon (*saṅku-chāya*); simple, quadratic, simultaneous and linear indeterminate equations (*kuṭṭaka*). In fact, the most significant contribution of Āryabhaṭa in the *Gaṇitapāda* is his method of solving the first-order indeterminate equation: to find solutions of $ax+by = c$, in integers (where a, b are given integers).

The *Kālakriyapāda*, the third part of the **Āryabhaṭiyam**, contains 25 verses explaining the various units of time and the method of determination of the positions of planets for a given day. Calculations concerning the *adhikamāsa* (intercalary month), *kṣayatithis*, the speed of planetary motions and the concept of weekdays are all included in this part of the text.

The *Golapāda* forms the fourth and the last part of the **Āryabhaṭiyam** and contains 50 stanzas. Important geometrical (and trigonometric)

aspects of the celestial sphere are discussed in the *Golapāda*. The important features of the ecliptic, the celestial equator, the node, the shape of the Earth, the cause of day and night, the rising of the zodiacal signs on the eastern horizon, etc., find a place in this last part of the text. In fact, much of the contents of the *Golapāda* of the **Āryabhaṭiyam** is generally discussed under the chapter called *triprasna* (three problems of time, place and direction) in the later siddhāntic texts. Another very important topic included in this chapter is the lunar and solar eclipses.

The system of astronomy expounded in the **Āryabhaṭiyam** is generally referred to as the *audāyika* system since the dawn of day is reckoned from the mean sunrise (*udaya*) at Laṅkā, a place on the Earth's equator. However, we learn from Varāhamihira and Brahmagupta that Āryabhaṭa I propounded another system of astronomy called *ārdha-rātrika* in which the day is reckoned from the mean midnight (*ārdha-rātri*) at Laṅkā. The important parameters are different in the two systems. However, Āryabhaṭa's text of the *ārdha-rātrika* system is not available now. Its outlines can be reconstructed from Brahmagupta's **Khaṇḍakhādyaka** and some later works.

The following are some of the innovative contributions of Āryabhaṭa I:

(1) A unique method of representing large numbers using the alphabets for the purposes of metrics and easy memorization. The method followed by Āryabhaṭa is different from the popular methods of *Kaṭapayādi* and *Bhūtasaṅkhyā* which also serve the same purpose. However, Āryabhaṭa's method was not followed by later astronomers due to the difficulty in pronouncing the words thus formed.

(2) The value of π is given as 3.1416, which is correct to the first four decimal places, for the first time in India. Āryabhaṭa gives the value of π as the ratio of 62,832 to 20,000. But he cautions that the value is "*āsanna*", i.e. approximate. The great Keralite astronomer, Nīlakaṇṭha Somayāji (1500 AD) provides the explanation that π is incommensurable (or irrational). This achievement of Āryabhaṭa I, as early as in the fifth century, is truly remarkable in view of the fact that it was only thirteen centuries later, in 1761, that Lambert proved that π is *irrational* (i.e. it cannot be expressed

as ratio of two integers). Again, it was almost about a century later, in 1882, that Lindemann established the fact that π is *transcendental*, i.e. it cannot be the root of an algebraic equation of any degree.

(3) Sine table: The importance of the trigonometric functions like sine (*jyā*) and cosine (*koṭijyā*) in Indian astronomy can hardly be exaggerated.

Āryabhaṭa I gives us the rules for the formation of the sine table in just one stanza. Accordingly, the sine values for the angles from 0° to 90° at intervals of 3°45′ can be obtained. The values, thus obtained, compare favourably with the modern values. It is important to note that for an angle ϑ, the "Indian-sine" (*Jyā*) of the angle ϑ is related to the modern sine by the relation:

$$\text{Jyā}\ (\vartheta) = R \sin \vartheta$$

where R is a pre-defined constant value of the radius of a circle. For example, Āryabhaṭa as well as the **Sūryasiddhānta** take the value $R = 3438'$, so that:

$$\text{Jyā}\ (\vartheta) = 3438' \sin \vartheta$$

Brahmagupta postulates that $R = 150$.

Āryabhaṭa also gives the following relations for the trigonometric ratios of "allied" angles like $90° + \vartheta$, $180° + \vartheta$ and $270° + \vartheta$:

(*i*) $\sin (90° + \vartheta) = \sin 90° - \text{versine}\ \vartheta = \cos \vartheta$

(*ii*) $\sin (180° + \vartheta) = \sin 90° - \text{versine}\ 90° - \sin \vartheta = -\sin \vartheta$

(*iii*) $\sin (270° + \vartheta) = (\sin 90° - \text{versine}\ 90°) - (\sin 90° - \text{versine}\ \vartheta)$
$= -\cos \vartheta$

where versine $\vartheta = 1 - \cos \vartheta$.

(4) Earth's shape and rotation: Now, it is a well-known fact that the Earth is spherical (or spheroidal) in shape and that it rotates about its own axis once a day causing day and night. He clearly maintains that:

(*i*) the Earth is spherical, circular in all directions (see *Gola-pāda*, 6).

(*ii*) Halves of the globes of the Earth and the planets are dark due to their own shadows; the other halves facing the Sun are bright. It is truly creditable that Āryabhaṭa recognised

that the Earth and the other planets are not self-luminous but receive and reflect light from the Sun.

(*iii*) Again, Āryabhaṭa was the first to state that the rising and setting of the Sun, the Moon and other luminaries are due to the relative motion caused by the rotation of the Earth about its own axis once a day. He says, "Just as a man in a boat moving forward sees the stationary objects (on either side of the river) as moving backward, just so are the stationary stars seen by the people at Laṅkā (i.e. on the equator) as moving exactly towards the west" (*Golapāda*, 9).

The period of one sidereal rotation (i.e., with reference to the fixed stars in the sky) of the Earth, as given by Āryabhaṭa works out to be 23h 56m 4.1s. The corresponding modern value is 23h 56m 4.091s. Thus, Āryabhaṭa's accuracy is truly remarkable.

1.6 Astronomers after Āryabhaṭa

Āryabhaṭa's cryptic and aphoristic style would have made it extremely difficult to understand his text, but for the detailed exposition of the system by Bhāskara I (c.600 AD). In his commentary on the **Āryabhaṭiyam**, as also in the works **Mahā**- and **Laghu-Bhāskarīyam**, Bhāskara I (to be distinguished from his more popular namesake of the 12th century) has very ably expounded Āryabhaṭa's astronomy with examples and references.

As mentioned earlier, Varāhamihira (505 AD) brought together the five systems of astronomy extant during his period, in his remarkable work, **Pañcasiddhāntikā**. He mentions that among the five systems, the **Sūryasiddhānta** is the best. Even to this day the most popular astronomical text is **Sūryasiddhānta**, though in its revised form. It is believed that the modern version of **Sūryasiddhānta** was composed around 1000 AD.

While the *siddhāntas* proper are large texts consisting of broad theories and a large number of topics, these texts are not very handy for practical computations and day-to-day use. Further, very large numbers will have to be dealt with, which is a very inconvenient task. Therefore, besides the *siddhāntas*, two other types of texts on astronomy have been used. These are called the *tantras* and *karaṇas*.

Conventionally, the *Siddhāntas* choose the beginning of the *Mahāyuga* (43,20,000 years) as the epoch. After the **Sūryasiddhānta,** two other popular *Siddhāntas* are **Brahmasphutasiddhānta** of Brahmagupta (628 AD) and **Siddhāntaśiromaṇi** of Bhāskara II (1150 AD). A large number of commentaries and even secondary commentaries have been written, particularly on the **Sūryasiddhānta.**

The *tantra* texts have comparatively fewer topics and explanations. These works choose the more convenient epoch, viz. the beginning of *Kaliyuga* – the sunrise of February 18,3102 BC. For example, the **Āryabhaṭiyam** and Nīlakaṇṭha Somāyaji's **Tantrasaṅgraha** (c.1500 AD) are *tantra* texts.

However, for practical computations and making *pañcāṅgas,* the most useful handbooks are the *karaṇa* texts. In these, practical algorithms are provided taking a convenient contemporary date as the epoch. The advantage of a recent epoch is that, one now deals with smaller numbers for the *ahargaṇa* (the number of civil days elapsed since the epoch). Further, since the corrected positions of planets for a recent date have been given with the necessary *bījasamskāra* (corrections), the computations based on these *karaṇa* handbooks are more accurate. The well known *karaṇa* texts are Brahmagupta's **Khaṇḍakhādyaka** (7th cent.), Bhāskara II's **Karaṇakutūhala** (12th cent.) and Ganeśa Daivajña's **Grahalāghava** (16th cent.). A large number of such handbooks and tables (*sāraṇīs*) were composed during different periods—even as late as the nineteenth century.

Some of the famous Indian astronomers and their major works are listed below. The dates in brackets refer to the approximate dates of composition of the works:

	Author		Works
(1)	Āryabhaṭa I	(499 AD)	**Āryabhaṭiyam, Āryasiddhānta**
(2)	Varāhamihira	(b.505 AD)	**Pañcasiddhāntikā, Bṛhatsamhitā**
(3)	Bhāskara I	(c.600 AD)	**Bhāṣhya** on **Āryabhaṭiyam, Mahābhāskarīyam, Laghubhāskarīyam**
(4)	Brahmagupta	(b.591 AD)	**Brahmasphuṭasiddhānta, Khaṇḍakhādyaka**
(5)	Vateśvara	(880 AD)	**Vateśvarasiddhānta**
(6)	Mañjula	(932 AD)	**Laghumānasam**

	Author		Works
(7)	Āryabhaṭa II	(950 AD)	**Mahāsiddhānta**
(8)	Bhāskara II	(b.1114 AD)	**Siddhāntaśiromaṇi, Karaṇakutūhala**
(9)	Parameśvara	(c.1400 AD)	**Dṛgganitam, Sūryasiddhānta-vivaraṇam, Bhaṭadīpikā,** etc.
(10)	Nīlakaṇṭha Somayāji	(1465 AD)	**Tantrasaṅgraha, Āryabhaṭabhāṣhya**
(11)	Ganeśa Daivajña	(1520 AD)	**Grahalāghava**
(12)	Jyeṣṭhadeva	(1540 AD)	**Yuktibhāṣā**
(13)	Candraśekhara	(b.1835 AD)	**Siddhāntadarpaṇa**
(14)	Śaṅkara Varman	(19th cent.)	**Sadratnamālā**
(15)	Veṅkateśa Ketkar	(1898 AD)	**Jyotirgaṇitam, Grahagaṇitam**

1.7 CONTENTS OF THE *SIDDHĀNTAS*

The various topics of interest in Indian astronomy are discussed in different chapters. A chapter is called an *adhyāya* or *adhikāra*. The distribution of the topics into the different *adhikāras* in a typical Siddhāntic text is given below.

1. *MADHYAMĀDHIKĀRA*

The word *madhyamā* means the average or 'mean' positions of planets. Here, by 'planets' we mean the Sun, the Moon and the so-called *tārāgrahas,* viz., Mercury, Venus, Mars, Jupiter and Saturn. In order to calculate the mean angular velocities, the numbers of revolutions completed in a *mahāyuga* (of 43,20,000 years) by the planets as well as the special points, viz., the apogee (called *mandocca*) of the Moon and also of the Moon's ascending node (popularly called *Rāhu*) are given.

The procedure to calculate the *ahargaṇa* (the number of civil days from the epoch) for the given date is also explained in this chapter. The total number of civil days in a *Mahāyuga* is also specified. Then, the motion of a planet from the epoch to the specified date is given by the equation:

Motion = (No. of revns. × *Ahargaṇa* × 360°) / (No. of civil days in a *Mahāyuga*)

in degrees.

When the nearest integral multiple of 360° (i.e., completed number of revolutions) is dropped from the above value, we get the *mean* position of the planet in degrees for the given date.

2. *SPAṢṬĀDHIKĀRA*

In this chapter the procedure to obtain the "true" position of a planet, from the mean position, is discussed. The word *spaṣṭa* means correct or true. For obtaining the true positions from the mean, two corrections are prescribed:

(*i*) *manda*, applicable to the Sun, the Moon and the other five planets and
(*ii*) *śīghra*, applicable only to the five planets (*tārāgrahas*), viz. *Budha, Śukra, Kuja, Guru* and *Śani*.

The *manda* correction takes into account the fact that the planet's orbits are not circular. This correction corresponds to what is called "the equation of the centre" in modern astronomy. The *śīghra* correction corresponds to the conversion of the heliocentric positions of planets to the geocentric positions.

3. *TRIPRAŚNĀDHIKĀRA*

This chapter deals with the "three questions" of direction (*dik*), place (*deśa*) and time (*kāla*). The procedures for finding the latitude of a place, the times of sunrise and sunset, variations of the points of sunrise and sunset along the eastern and western horizon, gnomon problems and calculation of *lagna* are dealt with therein.

4. *CANDRA- AND SŪRYA-GRAHAṆĀDHIKĀRA*

In these chapters the computations of the lunar and the solar eclipses are discussed. The instants of the beginnings, the middle and the ends, regions of visibility, possibility of the occurrence, totality, etc., of the eclipses are considered. Their computational procedures are elaborated.

In fact, for Indian astronomers, the true testing ground for the veracity of their theories and procedures depend very much on the successful and accurate predictions of eclipses. Of course, when minor deviations between computations and observations were noticed, necessary changes and corrections (*bīja samskāra*) were suggested.

Besides these four important topics, the Siddhāntic texts contain many other topics, which vary from text to text, like the first visibility of planets, the Moon's cusps, mathematical topics like *kuṭṭaka* (indeterminate equations), spherical trigonometry and the rationales of the formulae used.

1.8 Continuity in Astronomical Tradition

A characteristic feature of Indian astronomy is the unbroken continuity in the tradition, beginning from the Vedic period upto recent times. Starting from simple observation and a simple calendar, relevant to the contemporary needs during the Vedic times, there has been a gradual progress in the extent of astronomical topics considered, mathematical techniques developed, and refinement and sophistication in the computational algorithms, always aimed at greater accuracy during the Siddhāntic period of evolution, spread over nearly fifteen centuries.

The popularity of the existing *siddhānta* texts, like the **Sūrya-siddhānta**, are enhanced with elucidations and illustrations, by a large number of commentaries and secondary commentaries. For example, the **Āryabhaṭiyam** carries learned and exhaustive commentaries by Bhāskara I, Parameśvara and Nīlakaṇṭha Somāyaji among others. Pṛthū-dakasvāmin's commentary on the **Khaṇḍakhādyaka** of Brahmagupta, in addition to those by Bhaṭṭotpala and Āmarāja, is extremely useful. Bhāskara II has written his own commentary, **Vāsanā bhāshya,** on his magnum opus, **Siddhāntaśiromaṇi**. In fact, very often, the commentaries improve upon the parameters and computational techniques of the original texts and yield better results.

While Mañjula (932 AD) and Śrīpati (c. 1000 AD) introduced additional corrections for the Moon, Nīlakaṇṭha Somāyaji (c.1500 AD) revised the model of planetary motion in his **Tantrasaṅgraha** to obtain more precise positions of the interior planets, Budha and Śukra.

Inspired by the ideas of Parameśvara (c. 1400 AD), Nīlakaṇṭha (c. 1500 AD) developed a heliocentric model in which all planets move round the Sun in concentric orbits. It is a significant achievement before

Copernicus came into the picture. Nīlakaṇṭha's revised model was successfully adopted by all the later astronomers of Kerala, such as Jyeṣṭadeva, Acyuta Piśāraṭi and Citrabhānu.

It is also noteworthy that the knowledge of astronomy was not restricted to any particular region, but spread throughout India. While Candraśekhara Sāmanta of Orissa made quite a few important innovations, like an additional correction to the Moon quite independently, Kerala became the hub of tremendous development between the 14th and 19th centuries. Of course, the congenial social milieu and patronage must have played an important role in the development of astronomy—more during certain periods and in certain regions and less at other times and in other regions.

While concluding this historical review, it is necessary to remember that the methods described in this book are now far superseded by the methods of celestial mechanics which are based on Newton's theory of gravitation. But they form an important step in the advancement of astronomy, and contribute to our understanding of modern astronomy as a science.

2

Celestial Sphere

2.1 Introduction

On a clear night, when we look up at the sky we see a multitude of luminous heavenly bodies illuminating the sky. These luminous bodies appear to be shining from the inner surface of a large hollow hemisphere with the observer at the centre. Of course, we all know that the sky is not really a hollow crystalline sphere surrounding the Earth, although it appears like that. The stars, the planets, the Sun and the Moon are scattered through space at different and very large distances. All the same, the sky as a large hollow crystalline hemisphere with the Earth as its centre is a very convenient model for the study of the positions and motions of the celestial bodies. This hemispherical model of the sky is called the *Celestial sphere.* Thus, the celestial sphere is an imaginary hollow sphere of a very large radius with the Earth as its centre.

2.2 Diurnal Motion of the Celestial Bodies

The Earth rotates about its own axis from west to east in the course of a day. Due to this rotation of the Earth about its own axis, an observer on the surface of the Earth is carried eastward continuously. But the observer is not consciously aware of his motion in space. On the other hand, the celestial sphere with all the celestial bodies, as though clinging to it, appears to rotate from east to west. This apparent westward rotation of the celestial bodies is called their *diurnal motion.*

In fact, Āryabhaṭa I (47 AD) the famous Indian astronomer, mentioned explicitly in his **Āryabhaṭiyam** that:

(i) the Earth is round;

(ii) the Earth rotates about its own axis from west to east and hence all the celestial bodies appear to move in the opposite direction, from east to west, everyday.

Āryabhaṭa I gives a beautiful example of this relative motion. For an observer moving in a boat on a river, all the trees, etc. on the bank of that river appear to move in the opposite direction.

Due to the diurnal motion, the celestial bodies appear to rise in the east, move westward up in the sky and then set in the west. When the axis of the Earth's rotation is extended, it meets the celestial sphere at two diametrically opposite points called celestial poles. The one in the direction of the Earth's north pole is called the celestial north pole and the other pole, the celestial south pole (P and P' in Fig. 2.1). Due to the diurnal motion, every celestial body describes a circle whose plane is perpendicular to the line PP'.

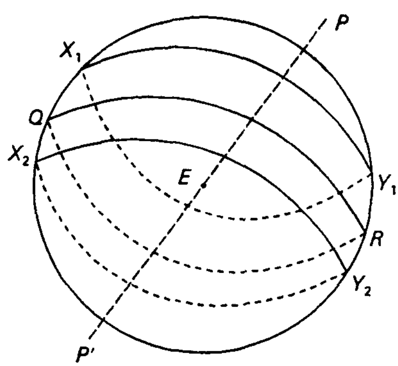

Figure 2.1 Celestial equator, poles and diurnal paths

The *great circle* of the celestial sphere is a circle on its surface with a radius equal to that of the sphere. The great circle (QR in Fig. 2.1), whose plane is perpendicular to the line PP' joining the celestial poles, is called the *celestial equator.* Thus, the diurnal paths of the celestial bodies are circles of different radii, parallel to the celestial equator. These are shown as X_1Y_1, X_2Y_2 etc. in Fig. 2.1.

2.3 Motion of Celestial Bodies Relative to Stars

In the previous section, we saw how all the celestial bodies appear to move from east to west due to the diurnal motion. This motion is common to all the stars, the planets and the Moon (note

that the Sun is a star). In addition to the diurnal motion, the planets, the Sun and the Moon have another, different motion. These are the actual individual motions of the said bodies and are observable relative to the stars and groups of stars (called constellations) which are fixed. Observations over long periods of time reveal how the different celestial bodies move, relative to the fixed stars.

The planets, the Sun and the Moon, as observed from the Earth, move relative to the fixed stars from *west to east*, the direction opposite to that of the diurnal motion. However, sometimes a planet may have "retrograde" motion (*vakra gati*); when it *appears* to move in the opposite direction, that is, from *east to west* relative to the fixed stars.

2.4 Celestial Horizon, Meridian

When an observer stands at a place on the surface of the Earth and looks around describing a full circle, he finds that the sky appears to meet the Earth along a circle. The circle, with the observer at the centre, is called the *celestial horizon* (*Kṣitija*). In fact, it is the great circle of the celestial sphere in which the tangent plane to the surface of the Earth, at the position of the observer, meets the celestial sphere. It is depicted by *SENW* in Fig. 2.2.

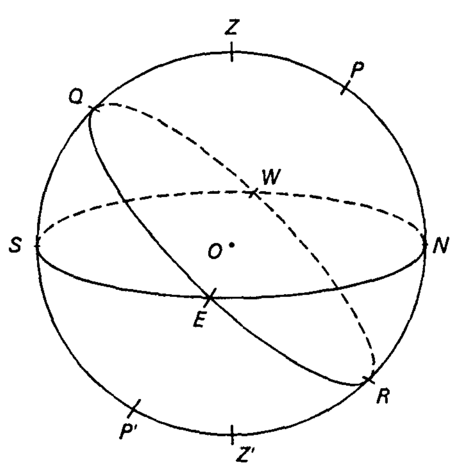

Figure 2.2 Celestial horizon and cardinal points

The line perpendicular to the plane of the horizon (plumb line), at the position of the observer, meets the celestial sphere at two points Z and Z' as shown in Fig. 2.2. These two points are respectively called the *zenith* and the *nadir*. Clearly, the *zenith* and the *nadir* are the poles of the celestial horizon. The great circle $PZP'Z'$ passing through the celestial poles and the *zenith* and the *nadir* is called the *celestial meridian* of the observer (see Fig. 2.2).

The celestial equator QR divides the celestial sphere into two hemispheres. The one containing the celestial north pole P is called the *northern* hemisphere and the other containing the celestial south pole P' is the *southern* hemisphere.

Similarly, the celestial meridian $PZP'Z'$ divides the celestial sphere into the *eastern* hemisphere and the *western* hemisphere.

The hemisphere "above" the celestial horizon, containing the zenith is called the *visible* hemisphere and the one "below" (containing the nadir) is the *invisible* hemisphere. The latter is so called since the celestial bodies in that hemisphere (below the horizon) cannot be seen. For example, during the day the Sun will be in the visible hemisphere (from sunrise to sunset) and during the night the Sun will be in the invisible hemisphere.

2.5 Pole Star and Directions

On a dark night, with a clear sky, how does one know the four directions? The directions can be recognized with the help of the stars in the sky. Of course, we see innumerable clusters of stars, some very bright, some others just visible and still other clusters rarely visible. After some time, all of them will have moved westward except one star which continues to be nearly in the same position even after a long time. This exceptional star is called the Pole Star (or *Dhruvanakṣatra*). *Dhruva*, in Sanskrit means one which is fixed or immovable. Although the pole star is not very bright, it can be located quite effortlessly. The direction of the pole star is very close to the *north*. This is so because the pole star is about 1° away from the celestial north pole. The direction, exactly opposite to that of the pole star, is very close to the *south*. The directions perpendicular to the line joining

the observer and the north point are the east and the west; i.e. when you stand facing the pole star, the direction to your right is the *east* and that to your left is the *west.* These points on the horizon, *N,S,E,W* in Fig. 2.2, are called *cardinal points.*

From time immemorial, navigators and other travellers have used the pole star as the guiding star in determining the directions during the night. However, a lay observer may mistake some other star for pole star. Then, is there a foolproof way of confirming the identification of this important star? Yes, during the months from March to September, around 9 o' clock in the night, one can see a group of seven stars in the shape of a hook or a question mark (?) in the northern portion of the sky, towards the northerly direction (see Fig. 2.3).

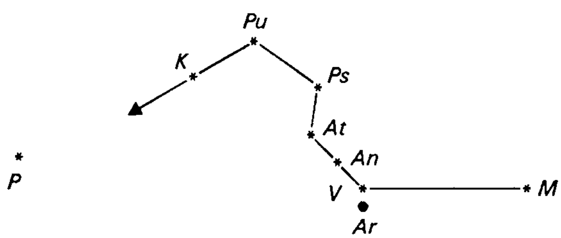

Figure 2.3 *Saptarṣi maṇḍala* and the pole star

This group of seven stars, all of more or less equal brightness, is called *Saptarṣi maṇḍala* in Indian astronomy. This cluster of seven stars is a "constellation" since the stars appear to be close to one another even though they may have different distances from us. The relative positions of the seven stars of the *Saptarṣi maṇḍala* are as shown in Fig. 2.3. The stars, taken in order, from one end are named as *Marīci* (*M*), *Vasiṣṭha* (*V*), *Aṅgīrasa* (*An*), *Atri* (*At*), *Pulastya* (*Ps*), *Pulaha* (*Pu*), *Kratu* (*K*). Very close to *Vasiṣṭha*, but slightly below it, there is a very faint star which is called *Arundhatī* (*Ar*). In Indian mythology, *Vasiṣṭha* and *Arundhatī* are an ideal couple. Even to this day in Hindu marriages, the stars *Vasiṣṭha* and *Arundhatī* are shown to newly-wed couples, suggesting that they should follow in the footsteps of this exemplary and ideal couple. The pole star (P) is in the direction joining the *Pulaha* (*Pu*) and *Kratu* (*K*) stars.

2.6 ZODIAC AND CONSTELLATIONS

We now briefly recapitulate the definitions given in the previous sections. Stars and planets, on a clear night, appear as luminous points set in the hemispherical dome of the sky. This imaginary sphere of an arbitrarily determined large radius is called the *celestial sphere* (*khagola*). This sphere has no real physical existence and, in fact, the stars and planets are at varying distances from the observer. Since the relative *angular* distances of the celestial bodies are of interest in spherical astronomy, their actual linear distances are not of that much importance.

In Fig. 2.4, *A* and *B* are two celestial bodies and *O* is the position of the observer which is taken as the centre of the celestial sphere.

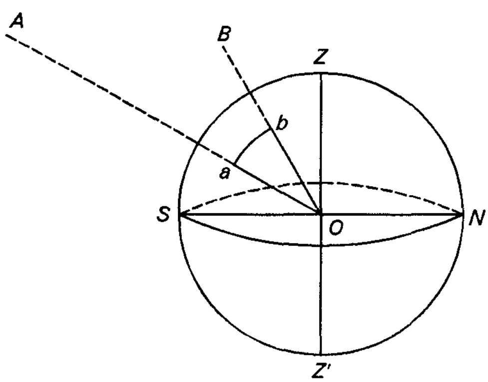

Figure 2.4 Celestial Sphere

The lines *OA* and *OB*, joining *A* and *B* to the observer's position *O*, cut the celestial sphere at the points *a* and *b* respectively. Angle $a\hat{O}b$ is the same as angle *AOB*, the *angular distance* between the celestial bodies *A* and *B* as seen from *O*. Thus, we observe that although the two objects *A* and *B* are at different distances from the observer, the angular distance between them remains the same, as though the two bodies lie on the celestial sphere with *O* as their centre.

The radius of the celestial sphere is taken arbitrarily as large, so that the entire Earth can be considered as just a point at the centre of this very huge, imaginary sphere. This means that wherever the observer may be on the surface of the Earth, he can always be considered as

being at the centre of the celestial sphere. However, it is important to note that all observers at different places on the Earth do not see the same part of the celestial sphere at a given time.

2.7 Equator and Poles (*Viṣuvad Vṛtta* and *Dhruva*)

The Earth is rotating about its own axis pp' as shown in Fig. 2.5. The axis pp' is extended both ways to meet the celestial sphere at P and P', which are called the celestial north and south poles (*uttara* and *dakṣiṇa dhruva*). The great circle qr on the Earth, whose plane is perpendicular to the axis pp' is the Earth's equator, and the points p and p' are the terrestrial north and south poles. Correspondingly, the great circle QR on the celestial sphere

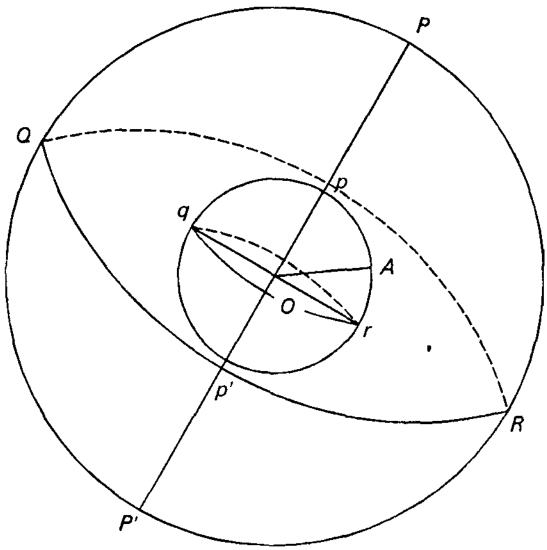

Figure 2.5 Celestial Equator and poles

is called the *celestial equator* (*viṣuvad vṛtta*). The points P and P' are the *celestial poles*. It is clear that the celestial equator QR is the intersection of the celestial sphere with the plane of the Earth's equator.

2.8 LATITUDE OF A PLACE AND ALTITUDE OF POLE STAR

The terrestrial (or geographical) latitude ϕ of a place on the surface of the Earth is the angular distance of the place from the Earth's equator. In other words, the latitude of a place is the angle made by the line joining the Earth's centre to the place with the plane of the Earth's equator. In Fig. 2.5, $A\hat{O}r$ is the latitude ϕ of the place A. The altitude of a celestial body is its angular distance from the horizon. To put it in ordinary language, the altitude of a luminary is the angle through which the observer has to raise his eyes, above the horizon, to see the body.

In Fig. 2.6, E is the centre of the Earth and O is the position of the observer on the Earth. The latitude ϕ of the observer O is the angle $O\hat{E}q$ made by the line EO with the plane of the terrestrial equator Eq. In Fig.2.6, r is the radius of the Earth.

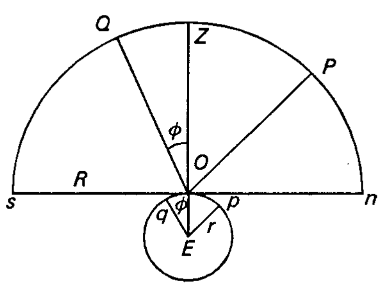

Figure 2.6 Altitude of the pole star and latitude of a place

The *altitude* of the pole star P is $n\hat{O}P$ (or arc nP), made by the line OP with the horizon sn of the place. In Fig. 2.6, $sQZPn$ represents the meridian on the celestial sphere with radius $R >> r$. Q, Z and P respectively, represent the equatorial point, zenith and celestial north pole. Now, we have

$$
\begin{aligned}
\text{Latitude } \phi &= O\hat{E}q = \text{arc } Oq \\
&= Q\hat{O}Z = \text{arc } QZ \\
&= \text{arc } PQ - \text{arc } PZ \\
&= 90^\circ - (\text{arc } Zn - \text{arc } Pn) \\
&= 90^\circ - (90^\circ - \text{arc } Pn) = \text{arc } Pn = \text{Altitude of } P
\end{aligned}
$$

Thus, the latitude of a place is equal to the altitude of the pole star at that place. Thus, for example, an observer in Chennai or Bangalore can locate the pole star at about 13° above the horizon.

2.9 Ecliptic and Equinoxes

The Sun appears to move around the Earth continuously, as seen from the Earth, from west to east with respect to the fixed stars, and comes back to the same position after a year. This motion of the Sun, for an observer on the Earth, is *apparent* and is a relative motion caused by the revolution of the Earth around the Sun in a year. The apparent annual path of the Sun round the Earth, with respect to the fixed stars is a great circle S_1S_2 (Fig. 2.7) called the *ecliptic*. The points of intersection of the ecliptic, S_1S_2, with the celestial equator QR are called *equinoxes* denoted by ♈ and ♎.

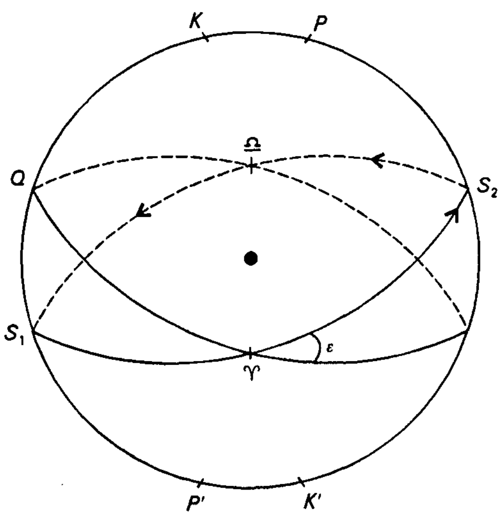

Figure 2.7 Ecliptic and Equinoxes

The equinoctial point ♈ where the Sun, during its annual motion along the ecliptic, crosses the celestial equator from the south to the north is called the *Vernal Equinox* or the first point of the *Aries* sign; and the other equinoctial point is called the *Autumnal Equinox* or the first point of the *Libra* sign. The angle between the planes of the ecliptic and the celestial equator is called *obliquity of the ecliptic*, denoted by ε. The value of ε is about $23°26'21''$ (as on 1-1-2000).

3

Co-ordinate Systems

3.1 Introduction

In order to determine and specify the location of a celestial body in the sky, different systems of co-ordinates are evolved. Each of these systems is useful in a particular context to locate a celestial body. The following are the different systems of co-ordinates:

1. Celestial longitude and latitude (or ecliptic system)
2. Right ascension and declination (or equatorial system)
3. Azimuth and altitude (or horizontal system)
4. Hour angle and declination (or meridian system)

We shall discuss these systems of co-ordinates in the following sections.

3.2 Celestial Longitude and Latitude (Ecliptic System)

Let S be the position of a body on the celestial sphere (Fig. 3.1). The ecliptic is represented by CL and its poles by K

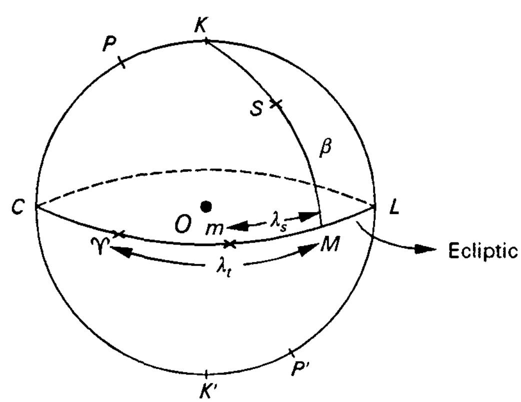

Figure 3.1 Celestial longitude and latitude

and K'. The arc KS is produced to meet the ecliptic CL and M. The angular distance SM (equal to angle SOM) of the body from the ecliptic is called the *celestial latitude* of the body S, and is denoted by the letter β. The latitude is northerly if S lies on the same side of the ecliptic as the north pole P and negative otherwise. Of course, if S is on the ecliptic, the latitude $\beta = 0$.

The angular distance ΥM, measured eastward along the ecliptic from the vernal equinox Υ or the first point of Aries is called the *celestial longitude* of the planet S and is denoted by λ_t, it varies from 0° to 360°.

The longitude λ_t measured from the vernal equinox Υ is called tropical longitude. However, in the Indian system the longitude is measured from a fixed point (see in Fig. 3.1) on the ecliptic, called *Meṣādi*. It is called the sidereal or *nirayana* longitude and it will be denoted by λ_s. The difference $\delta_p = \lambda_t - \lambda_s$ is called *ayanāmśa* which arises on account of the phenomenon of precession of the equinoxes discussed in Section 3.6.

3.3 Right Ascension and Declination (Equatorial System)

In this system the celestial equator QR is the great circle of reference and the first point of Aries is the origin.

In Fig. 3.2, let S be a celestial body and M be the foot of the secondary line through it to the celestial equator. Now, ΥM measured along

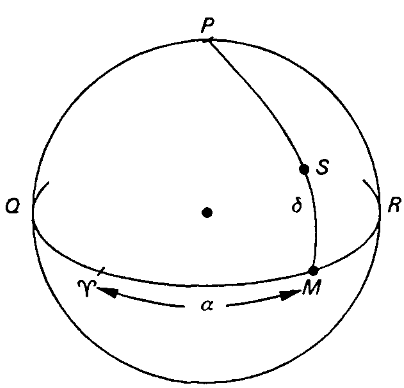

Figure 3.2 Right ascension and declination

the celestial equator is called the *right ascension* (R.A.) denoted by α, and SM is called the *declination* δ of the body S. The right ascension is measured from 0° to 360°. But, usually the R.A. of a heavenly body is expressed in time units as hours, minutes and seconds (by dividing the angle by 15° to get hours, etc.).

When a body is in the northern hemisphere of the celestial sphere, its declination is said to be north and when the body is in the southern hemisphere, it is south.

In the course of the Sun's motion in a year, its declination δ increases from 0° (March 22) to about 23°27′N (June 22), then decreases from 23°27′N to 0° (Sept 23). For the next six months, the Sun will go "south" of the celestial equator; δ increases from 0° (Sept 23) to 23°27′S (Dec. 22) and then it increases from 23°27′S to 0° (March 22). The northern course (Dec. 22 to June 22) and the southern course (June 22 to Dec. 22) of the Sun are referred to as the *Uttarāyaṇa* and the *Dakṣināyana*. However, the difference in the prevailing wrong dates of observance (viz. Jan. 14 or 15 and July 16) is due to the precession of the equinoxes, discussed in Section 3.6.

3.4 Azimuth and Altitude (Horizontal System)

In this system, the celestial horizon is the reference circle and the north point is the origin. In Fig. 3.3, let S be a celestial body and M the foot of the "vertical" through S.

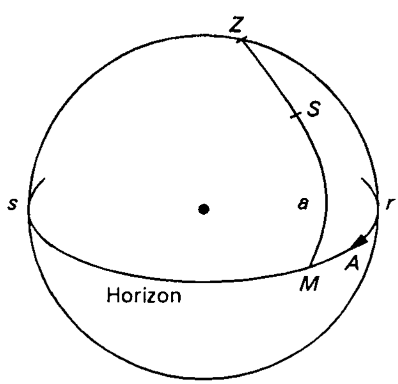

Figure 3.3 Azimuth and altitude

The angular distance *nM* from the north point *n*, measured along the horizon, is called the *azimuth* of the body *S*. The angular distance *SM* along the vertical through *S* is called the *altitude* of *S*. The azimuth *A* is always measured eastward from the north point *n* and varies from 0° to 360°. Sometimes the azimuth is also measured westward, in which case a specific mention is made to that effect.

The altitude *a* of a body is the angular distance from the horizon measured along the vertical through the body. It varies from 0° to 90° on either side of the horizon.

The horizontal system of azimuth and altitude is suitable for local, short interval observations. These co-ordinates are affected by diurnal motion and also vary from one place of observation to another.

3.5 Hour Angle and Declination (Meridian System)

Here, the celestial equator is the reference circle and the visible point of intersection is *Q* in Fig. 3.4 of the meridian *PQs*, with the equator as the origin. Let *S* be a celestial body and *M* be the foot of the secondary to the celestial equator through *S*. As defined earlier (Section 3.3), the angular distance *SM* from the celestial equator, along the secondary through *S*, is the *declination* δ of the body *S*.

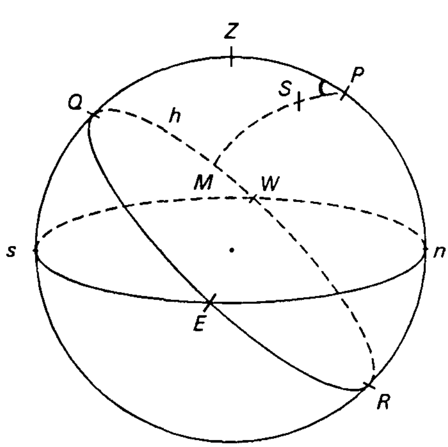

Figure 3.4 Hour angle and declination

The angular distance $Q\hat{P}M (= \text{arc } QM)$ is called the *hour angle h* of the celestial body *S*. Thus, the hour angle of *S* at any given instant is the angle between the meridian of the place and the hour circle *PSM* through *S*. The hour angle *h* is always measured westward from the meridian. However, in some contexts, if *h* is measured eastward, then it will be referred to as the eastern hour angle. The hour angle varies from 0° to 360°.

While the hour angle is affected by the diurnal motion, the declination δ is not affected. Further, *h* changes from one place of observation to another.

3.6 Phenomenon of Precession of Equinoxes

It is found that the equinoctial points ♈ and ♎, the points of intersection of the celestial equator and the ecliptic, are not fixed. In fact, these two points retrograde, i.e. move backwards along the ecliptic due to the rotation of the axis of the Earth around the axis joining the ecliptic poles. This phenomenon is called the "precession of the equinoxes" and it is important to know this to appreciate the difference between the frameworks of the western and the Indian astronomical systems.

In Fig. 3.5, let *S* be the position of a celestial body on the celestial sphere. Then SS_1 is the celestial latitude of *S* measured along the great

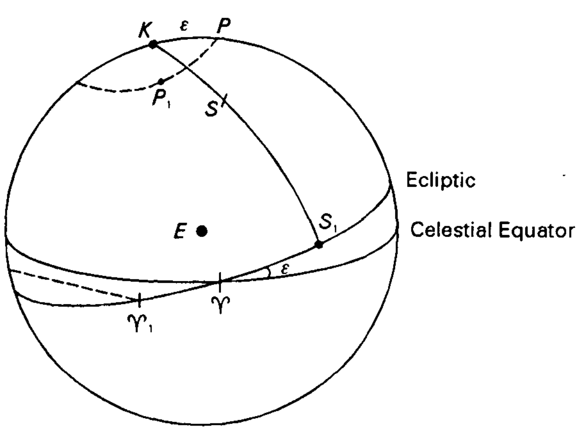

Figure 3.5 Precession of equinoxes

circle KSS_1, passing through the pole K of the ecliptic. The longitude of S is ΥS_1 which is measured from Υ along the ecliptic.

The ecliptic is a fixed, great circle on the celestial sphere with reference to the background of the stars. The celestial equator keeps on moving, though slowly, in such a way that the first point of Aries Υ moves backwards (retrogrades) along the ecliptic, say to a position Υ_1 at a future time, at an average annual rate of $50''.27$.

Further, if the change in the *obliquity* of the ecliptic ε is considered as being very negligible, the celestial north pole P of the celestial equator describes a small circle of which the pole is K and the angular radius is ε. It is clear from Fig. 3.5 that $\Upsilon\Upsilon_1$ is equal to the angle PKP_1 which gives the distance that the equinoctial point Υ has moved, P_1 being the pole of the changed position of the celestial equator (shown with a broken line, $Q\Upsilon_1$ arc, in Fig. 3.5) which intersects the ecliptic at Υ_1.

3.7 Ancient Indian References to the Precession

The phenomenon of the precession of the equinoxes appears to have been familiar to Indian astronomers since the Vedic times, and they used it for correcting their calendar.

Next, comes the question of the so-called "zero-year" when the zero of the Indian zodiac, considered as a fixed one, coincided with the zero of the moving zodiac, viz. the vernal equinox. In other words, when was the last time the first point of *Meṣa* coincided with the first point of Aries?

Indian astronomers have differed on the rates of precession during different periods, as also with respect to the "zero year". The **Sūrya-siddhānta** takes the rate of precession as $54''$ per year. The accumulated amount of precession starting from the "zero-year" is called *ayanāmśa*.

The zero-year according to different Indian astronomical texts is given below:

Siddhāntic text	**Year of zero ayanāmśa**
Sūryasiddhānta	499 AD
Laghumānasam (Mañjula)	527 AD
Grahalāghava (Ganesá Daivajña)	522 AD
Bhaṭatulya (Dāmodara)	420 AD

3.8 Effects of Precession on Celestial Longitude

The vernal equinox has a precession rate of about 50″.27 per year. Therefore, it takes about 71.7 years for ♈ to move by 1°. The equinox completes one full revolution, moving backwards, along the ecliptic, in about 25,800 years. Since the celestial longitudes of all the heavenly bodies are measured along the ecliptic, starting from the vernal equinox ♈ and using it as the reference point and this point of reference itself moves backward along the ecliptic, the celestial longitudes of all these bodies increase by a constant amount of about 50″.27 each year. However, the latitudes are unaffected by the precession of the equinoxes since the plane of the ecliptic remains fixed.

3.9 Tropical (*Sāyana*) and Sidereal (*Nirayana*) Longitudes

In modern astronomy, the zodiac is divided into twelve signs, viz. Aries, Taurus, etc. starting from the vernal equinoctial point called "the first point of Aries" as noted earlier. The celestial longitudes of heavenly bodies measured along such a "moving zodiac" are referred to as tropical (or *sāyana*) longitudes. In Sanskrit *sāyana* means "with motion". However, as pointed out earlier, in Indian astronomy, the celestial longitudes are measured starting from a fixed point of reference called *Meṣādi.* Thus, a "fixed zodiac" is used in this system where the celestial longitudes are measured with reference to the fixed stars. Hence, the longitudes are referred to as *sidereal* (or *nirayana*); *nirayana* means "without motion".

Now, due to precession, the first point of Aries moves backwards constantly as compared to the first point of *Meṣa*. Once in about 25,800 years, the period of the complete revolution of ♈, the first points of the moving and the fixed zodiacs coincide when the longitudes, according to the *sāyana* and the *nirayana* systems, will be the same.

However, there is a divergence of opinion as to when ♈ coincided with the first point of *Meṣa* last – the year of "zero precession". On the recommendation of the Indian Calendar Reform Committee, the Government of India has adopted 285 AD as the year of zero-precession.

4

Rāśi and *Nakṣatra* Systems

4.1 Zodiac and *Rāśis*

Consider two small circles parallel to the ecliptic and lying at an angular distance of 8° on either side of the ecliptic. The positions of stars and planets are considered with reference to this circular belt, called the *zodiac* (*bhacakra*).

The zodiac is divided into 12 equal parts, each part of 30° extent, called *signs* (*sāyana rāśi*) and which are listed in Table 4.1. Each sign (or *sāyana rāśi*) was once characterized by groups of stars called constellations. These are named after the objects or animals or human forms which they are supposed to resemble. The twelve groups of stars, characterizing the twelve signs, are called *zodiacal constellations.*

The *sāyana rāśis* have now shifted from the constellations, after which those were named, by the 'phenomenon of precession of equinoxes'. The shift now amounts to about 23°45′. In India we use the unshifted *nirayana rāśis.*

The Sun moves from one sign to the next in the course of a month. It is at the first point of Aries, i.e. at the vernal equinox, around March 22nd and at the first point of Libra, i.e. at the autumnal equinox, around September 23rd each year.

For example, on January 1st, 1990 at 5:30 a.m. (IST), the tropical longitude λ_t of the sign was 280°18′. This means that the Sun was at 10°18′ in the sign of Capricorn (*Makara*) whose range is 270° to 300°. However, in the Indian system its sidereal longitude is 23°43′ less, i.e. $\lambda_s = 250°35'$, which falls in Sagittarius (*Dhanus*).

Table 4.1 Signs of the zodiac

	Signs	*Sāyana rāśis*	Meaning	Angle of lat.
1.	Aries	*Meṣa*	Ram	0° – 30°
2.	Taurus	*Vṛṣabha*	Bull	30° – 60°
3.	Gemini	*Mithuna*	Twins	60° – 90°
4.	Cancer	*Karkaṭaka*	Crab	90° – 120°
5.	Leo	*Simha*	Lion	120° – 150°
6.	Virgo	*Kanyā*	Virgin	150° – 180°
7.	Libra	*Tulā*	Balance	180° – 210°
8.	Scorpio	*Vṛścika*	Scorpion	210° – 240°
9.	Sagittarius	*Dhanus*	Archer	240° – 270°
10.	Capricorn	*Makara*	Sea goat	270° – 300°
11.	Aquarius	*Kumbha*	Water carrier	300° – 330°
12.	Pisces	*Mīna*	Fish	330° – 360°

Note: The signs and *rāśis* shown in the first two columns of the table were equivalent when the first point of Aries (i.e. the vernal equinox) coincided with the first point of *Meṣa* of the Indian system.

4.2 *Nakṣatra* System

The zodiac is divided into 27 equal parts called *nakṣatras*, this division has been followed from ancient times. Each *nakṣatra* is of 13°20′ (=360°/27) extent. The *nakṣatras* are calculated from the first point of *Meṣa*, the starting point of the zodiac in the Indian system. Thus, the twelve *nirayana rāśis* are equivalent to the twenty-seven *nakṣatras* in the zodiac so that each *rāśi*, in its angular extent, is equivalent to two and a quarter *nakṣatras*. For example, the nirayana *Meṣa rāśi* constitutes two full *nakṣatras*, *Aśvinī* and *Bharaṇī* and a quarter (*pāda*) of the *nakṣatra*, *Kṛttikā* (see Table 4.3).

Each *nakṣatra* is subdivided into 4 equal parts, each part being called a *pāda*. Thus, totally 108 (=27 × 4) *nakṣatra pādas* constitute the zodiac. These 108 *pādas* are equally distributed into 12 *rāśis* so that each *rāśi* consists of 9 *pādas*.

The *nakṣatras* are also called "lunar mansions" since the Moon covers these 27 *nakṣatras* in the course of a sidereal month, i.e., the time taken by the Moon to complete a revolution around the Earth with respect to the fixed stars. The average length of a sidereal month is 27.3217 days.

The following table (Table 4.2) gives the list of the 27 *nakṣatras*, in their natural order, and their angular extents. Each *nakṣatra* was named after the most prominently visible star (called *yogatārā* or junction-star) contained within its range, given in Table 4.2. The *yogatārās* with modern equivalents and their co-ordinates are listed in Table 4.3 (reproduced from *Lahiri's Ephemeris* for 1995).

The distribution of the *pādas* (quarters) of the 27 *nakṣatras* into 12 *rāśis* is shown in Table 4.4.

Table 4.2 *Nakṣatras* and their range of *nirayana* longitudes

No.	*Nakṣatra*	From	To
1.	*Aświnī*	0°0′	13°20′
2.	*Bharaṇī*	13°20′	26°40′
3.	*Kṛttikā*	26°40′	40°00′
4.	*Rohinī*	40°00′	53°20′
5.	*Mṛgaśira*	53°20′	66°40′
6.	*Ārudrā*	66°40′	80°00′
7.	*Punarvasu*	80°00′	93°20′
8.	*Puṣya*	93°20′	106°40′
9.	*Āśleṣā*	106°40′	120°00′
10.	*Makhā* (or *Maghā*)	120°00′	133°20′
11.	*Pubba* (*Pūrva Phālgunī*)	133°20′	146°40′
12.	*Uttarā* (*Uttarā Phālgunī*)	146°40′	160°00′
13.	*Hasta*	160°00′	173°20′
14.	*Cittā* (or *Citrā*)	173°20′	186°40′
15.	*Svātī*	186°40′	200°00′
16.	*Viśākhā*	200°00′	213°20′
17.	*Anurādhā*	213°20′	226°40′
18.	*Jyeṣṭhā*	226°40′	240°00′
19.	*Mūlā*	240°00′	253°20′
20.	*Pūrvāṣādhā*	253°20′	266°40′
21.	*Uttarāṣādhā*	266°40′	280°00′
22.	*Śravaṇa*	280°00′	293°20′
23.	*Dhaniṣṭhā*	293°20′	306°40′
24.	*Śatabiṣaj*	306°40′	320°00′
25.	*Pūrvābhādrā*	320°00′	333°20′
26.	*Uttarābhādrā*	333°20′	346°40′
27.	*Revatī*	346°40′	360°00′

Table 4.3 Mean places of stars for 1995.0 (i.e. on Jan. 0.822 UT = Jan. 0. 25h 14m IST)
Note: Tropical or *Sāyana* long. = *Nirayana* long. +23°47′14″.1 viz. Mean *Ayanāmśa*

Star	Indian Name	Mag.	*Nirayana* Longitude				Latitude			Right Ascension			Declination		
			s	°	′	″	°	′	″	h	m	s	°	′	″
β Arietis	*Aśvinī*	2.72	0	10	06	46	+8	29	14	1	54	21.8	+20	47	01
∝ Arietis	...	2.23	0	13	48	19	+9	57	54	2	06	53.4	+23	26	20
41 Arietis	*Bharaṇī*	3.68	0	24	20	48	+10	26	58	2	49	41.3	+27	14	24
Algol 1	...	2.7v	1	2	18	39	+22	25	41	3	07	50.5	+40	56	12
Alcyone 2	*Kṛttikā*	2.96	1	6	08	07	+4	03	02	3	47	11.2	+24	05	24
Aldebaran 3	*Rohinī*	1.06	1	15	55	56	−5	28	04	4	35	38.0	+16	29	58
Rigel 4	...	0.34	1	22	58	21	−31	07	24	5	14	17.8	−8	12	26
Bellatrix 5	...	1.70	1	27	05	22	−16	49	00	5	24	51.8	+6	20	44
Capella 6	*Brahmahṛday*	0.21	1	28	00	03	+22	51	51	5	16	19.1	+45	59	36
β Tauri	*Agni*	1.78	1	28	43	05	+5	23	05	5	25	58.5	+28	36	13
ε Orionis	...	1.75	1	29	36	24	−24	30	25	5	35	57.5	−1	12	17
λ Orionis	*Mṛgaśiras*	3.66	1	29	50	59	−13	22	12	5	34	51.7	+9	55	52
Polaris	*Dhruva*	2.lv	2	4	42	39	+66	06	03	2	26	21.9	+89	14	31
Betelguese 7	*Ārdrā*	0.6v	2	4	53	51	−16	01	40	5	54	54.1	+7	24	23
Sirius 8	*Lubdhaka*	−1.58	2	20	13	32	−39	36	15	6	44	55.6	−16	42	32
Canopus 9	*Agastya*	−0.86	2	21	06	17	−75	49	28	6	23	50.6	−52	41	34
Castor 10	...	1.99	2	26	23	02	+10	05	44	7	34	16.9	+31	53	59
Pollux 11	*Punarvasu*	1.21	2	29	21	34	+6	41	02	7	45	00.6	+28	02	19
Procyon 12	...	0.48	3	1	55	46	−16	01	07	7	39	01.7	+5	14	17
δ Cancri	*Puṣya*	4.17	3	14	51	54	+0	04	37	8	44	24.1	+18	10	23

Continues

Star	Indian Name	Mag.	*Nirayana* Longitude				Latitude			Right Ascension			Declination		
			s	°	′	″	°	′	″	h	m	s	°	′	″
ε Hydrae	*Āśleṣā*	3.48	3	18	29	18	−11	06	15	8	46	30.7	+6	26	14
α Cancri	”	4.27	3	19	47	05	−5	04	51	8	58	12.8	+11	52	38
Dubhe 13	*Kratu*	1.95	3	21	20	25	+49	40	47	11	03	25.3	+61	46	41
Regulus 14	*Makhā*	1.34	4	5	58	21	+0	27	53	10	08	06.3	+11	59	30
δ Leonis	*Pūrva Phālgunī*	2.58	4	17	27	33	+14	20	00	11	13	50.6	+20	33	04
Denebola 15	*Uttara Phālgunī*	2.23	4	27	45	39	+12	16	02	11	48	48.3	+14	36	00
δ Corvi	*Hasta*	3.11	5	19	35	42	−12	11	45	12	29	36.3	−16	29	16
Spica 16	*Citrā*	1.21	5	29	59	03	−2	03	15	13	24	35.7	−11	08	07
Arcturus 17	*Svātī*	0.24	6	0	22	36	+30	44	23	14	15	26.0	+19	12	30
α Libra	*Viśākhā*	2.90	6	21	13	32	+0	20	01	14	50	36.1	−16	01	16
β Centauri	...	0.86	6	29	56	09	−44	08	13	14	03	28.0	−60	20	57
α Centauri	...	0.06	7	5	37	45	−42	35	39	14	39	15.3	−60	48	54
δ Scorpii	*Anurādhā*	2.54	7	8	42	50	−1	59	08	16	00	02.2	−22	36	28
Antares 18	*Jyeṣṭhā*	1.2v	7	15	54	19	−4	34	09	16	29	06.0	−26	25	16
λ Scorpii	*Mūlā*	1.71	8	0	43	43	−13	47	16	17	33	16.1	−37	06	02
δ Sagittarii	*Pūrvāṣāḍhā*	2.84	8	10	43	26	−6	28	18	18	20	40.5	−29	49	52
ε Sagittarii	...	1.95	8	11	13	18	−11	03	04	18	23	50.4	−34	23	14
δ Sagittarii	*Uttarāṣāḍhā*	2.14	8	18	31	41	−3	26	56	18	54	57.3	−26	18	12
Vega 19	*Abhijit*	0.14	8	21	27	31	+61	43	59	18	36	46.1	+38	46	44
Altair 20	*Śravaṇa*	0.89	9	7	55	06	+29	18	13	19	50	32.4	+8	51	18
β Capricorni	...	3.25	9	10	11	25	+4	35	21	20	20	43.8	−14	47	51

Continues

Star	Indian Name	Mag.	*Nirayana* Longitude				Latitude			Right Ascension			Declination		
			s	°	′	″	°	′	″	h	m	s	°	′	″
β Delphini	*Dhaniṣṭhā*	3.72	9	22	29	04	+31	55	07	20	37	18.9	+14	34	39
α Delphini	...	3.86	9	23	31	25	+33	01	22	20	39	24.4	+15	53	39
Formalhaut 21	...	1.29	10	10	00	10	–21	08	06	22	57	22.5	–29	38	36
Deneb 22	...	1.33	10	11	28	23	+59	54	24	20	41	15.7	+45	15	44
λ Aquarii	*Śatabhiṣaj*	3.84	10	17	43	08	–0	23	11	22	52	21.2	–7	36	23
Achernar 23	...	0.60	10	21	27	12	–59	22	41	1	37	31.7	–57	15	43
Markab 24	*Pūrvābhādrapada*	2.57	10	29	37	43	+19	24	22	23	04	30.7	+15	10	42
β Pegasi	...	2.6v	11	5	31	02	+31	08	26	23	03	31.9	+28	03	20
γ Pegasi	*Uttarābhādrapada*	2.87	11	15	17	57	+12	36	00	0	12	58.7	+15	09	21
α Andromeda	...	2.15	11	20	27	06	+25	40	50	0	08	07.7	+29	03	45
ζ Piscium	*Revatī*	5.57	11	26	01	13	–0	12	48	1	13	28.2	+7	32	56

1. β Persei	7. α Orionis	13. α Ursae Majoris	19. α Lyrae
2. η Tauri	8. α Canis Majoris	14. α Leonis	20. α Aquilae
3. α Tauri	9. α Carinae	15. β Leonis	21. α Piscis Austrini
4. β Orionis	10. β Germinorum	16. α Virginis	22. α Cygni
5. γ Orionis	11. α Germinorum	17. α Bootis	23. α Eridani
6. α Aurigae	12. α Canis Minoris	18. α Scorpii	24. α Pegasi

Table 4.4 Distribution of *Nakṣatra Pādās* into *Rāśis*

No.	*Nirayana Rāśi*	*Nakṣatra*	*Pādās* included
1.	*Meṣa*	*Aśvinī*	All
		Bharaṇī	All
		Kṛttikā	1
2.	*Vṛṣabha*	*Kṛttikā*	2,3,4
		Rohinī	All
		Mṛgaśira	1,2
3.	*Mithuna*	*Mṛgaśira*	3,4
		Ārudrā	All
		Punarvasu	1,2,3
4.	*Karkaṭaka*	*Punarvasu*	4
		Puṣya	All
		Āśleṣā	All
5.	*Simha*	*Makhā*	All
		Pubba	All
		Uttarā	1
6.	*Kanyā*	*Uttarā*	2,3,4
		Hasta	All
		Cittā (Citrā)	1,2
7.	*Tulā*	*Cittā (Citrā)*	3,4
		Svātī	All
		Viśākhā	1,2,3
8.	*Vṛścika*	*Viśākhā*	4
		Anurādhā	All
		Jyeṣṭhā	All
9.	*Dhanus*	*Mūlā*	All
		Pūrvāṣāḍhā	All
		Uttarāṣādhā	1
10.	*Makara*	*Uttarāṣādhā*	2,3,4
		Śravaṇa	All
		Dhaniṣṭhā	1,2
11.	*Kumbha*	*Dhaniṣṭhā*	3,4
		Śatabhiṣaj	All
		Pūrvābhādrā	1,2,3
12.	*Mīna*	*Pūrvābhādrā*	4
		Uttarābhādrā	All
		Revatī	All

5

Time in Indian Astronomy

5.1 Introduction

The concept of time (*kāla*) is explained very systematically in Indian astronomy. It is significant that time has been considered both at the microcosmic and the macrocosmic levels. Depending on the needs of particular topics in astronomy, different scales and units of time are used.

On the macrocosmic scale, the *yuga* system is evolved as a theoretical model for describing motions of planets. On the microcosmic scale, as small a unit of time as *truṭi* ($^{1}/_{33750}$ of a second) has been mentioned by Bhāskara II.

However, based on periodical natural observations such as the sunrise and sunset, new moon, full moon and seasons, a working scale of time—consisting of day, fortnight, month and year—has been used from early times.

5.2 Civil Day and Sidereal Day

We saw earlier, how, due to diurnal motion, the Sun rises in the eastern horizon, moves up in the sky westward and sets in the western horizon. Then, from the sunset to the next sunrise, it will be below the horizon during the night. The duration between two successive risings of the Sun is called a *civil day* (or *sāvana dina*). Observations of sunrise over a very long time has revealed that the duration of a day is not constant but varies from day to day although slightly. This average solar day or mean solar day is referred to as a mean civil day (or *madhya sāvana dina*). This *sāvana dina* is divided into 60 equal parts called ghatikas or 24 equal parts called *horā* (hours).

The time taken by the fixed stars to go around is called a *sidereal day* (*nākṣatra dina*); it is equal to the period of the rotation of the Earth. The word "sidereal" is used as a reference to stars. It is important to

note that the time taken by the fixed stars to go round the Earth once is not the same as that taken by the Sun. While all the celestial bodies appear to move from east to west due to the diurnal motion, the Sun would have moved from west to east along the ecliptic by about 1°, relative to the stars. Therefore, the fixed stars take a little less than 24 hours (mean civil day) to complete a rotation round the Earth. In other words, a sidereal day is slightly less than a mean civil day. As a natural consequence of this, if a particular star rises in the eastern horizon at a particular time today, it will rise about two hours earlier after 30 days, 4 hours earlier after 60 days, and so on.

It is found that the stars take about 3 minutes and 56 seconds less than the Sun to go round the Earth once. In other words,

$$\begin{aligned} 1 \text{ civil day} &= 24 \text{ hours} \\ \text{and } 1 \text{ sidereal day} &= 23^h\, 56^m\, 4^s. \end{aligned}$$

5.3 Solar Year and Civil Calendar

The time taken by the Sun to complete a revolution around the Earth, as observed from the Earth, is defined as a *solar year.* But then, depending on the points of reference chosen, we have different types of solar years. Here, we consider only two types of solar years, viz. the *sidereal* (or *nirayana*) solar year and the *tropical* (or *sāyana*) solar year.

A *sidereal* solar year is the time taken by the Sun to complete a revolution with reference to a fixed star. Careful observations carried out over a long time enabled the ancient Indian astronomers to determine the duration of a sidereal year as 365.256364 days (or 365 days, 6 hours, 9 minutes, 9.8 seconds). Thus, we have the equation, 1 sidereal year = 365.256364 days.

On the other hand, the time taken by the Sun to complete a revolution along the ecliptic with reference to the vernal equinox ♈ is called a *tropical* solar year (*sāyana sauravarṣa*). Now, 1 tropical year is equal to 365.242190 days (or 365 days, 5 hours, 48 minutes, 46 seconds). It is the tropical year that determines seasons.

While discussing the difference between a civil day and a sidereal day we observed that, while the Sun completes a diurnal revolution in a civil day, the fixed stars complete a little more than a full diurnal revolution in the same duration of time. After each revolution of the Sun, this extra-angular motion of the fixed stars keeps accumulating.

After T mean solar days (or about 365.25 civil days), while the Sun would have made T revolutions, with reference to the fixed stars, the stars themselves will have completed $(T+1)$, i.e., about 366.25 revolutions around the Earth. In other words, approximately,

$$T \text{ mean solar days} = (T+1) \text{ sidereal days.}$$

Therefore, approximately

$$\begin{aligned} 1 \text{ sidereal day} &= T/(T+1) = 365.25 / 366.25 \text{ civil day} \\ &= 365.25 \times 24 / (366.25) \text{ hours} \\ \text{i.e., } 1 \text{ sidereal day} &= 23\text{h } 56\text{m } 4.1\text{s} \end{aligned}$$

5.4 Solar Month and Lunar Month

The angular extent of the ecliptic is 360° and the Sun takes about 365.25 civil days to complete one revolution. A solar month is the time taken by the Sun to cover a *rāśi* (of 30°) along the ecliptic. Since there are 12 *rāśis* on the ecliptic, a solar month is equal to about 365.25 / 12 days. To be precise, we have

$$\begin{aligned} 1 \text{ mean sidereal solar month} &= 365.256364 / 12 \text{ days} \\ &= 30.43803 \text{ days.} \end{aligned}$$

The Moon, like the Sun, moves from west to east with reference to the fixed stars. The motion of the Moon is much faster than that of the Sun. The Moon moves along its own orbit, inclining towards the ecliptic at an angle of about 5°8′. Since this angle is small, the Moon can be considered as moving along the ecliptic itself. It is found that the Moon takes an average period of 27.3216615 days to complete a revolution with reference to the fixed stars. This time interval is called the *sidereal period* of the Moon, since the Moon takes this much time

to cover an angular extent of 360°, as its motion per day is given as 360°/27.3216615, i.e. about 13°10′. In comparison to this, the Sun's motion along the ecliptic, in the same direction, is about 1° per day. Thus, the Moon overtakes the Sun by about 12°10′ per day.

Now, if the Sun and the Moon are moving in the same direction, in which case their celestial longitude is the same, as seen from the Earth, the Moon is said to be "new" or it is called a *new moon day.* After 24 hours, the Moon will have moved ahead of the Sun by about 12°10′. This separation of the Moon from the Sun goes on increasing at the rate of 12°10′ per day. When it becomes 360°, the Moon will again be in the same direction ("conjunction") as the Sun, resulting in a new moon. It is found that the current average interval between two successive new moons is 29.530589 days (29d, 12h, 44m, 2.9s).

The interval between two successive new moons is called the *synodic period* of the Moon ("synodic" means successive conjunctions of the same bodies). This synodic period of the Moon is defined as a *lunar month* (*candra māsa*). Thus,

$$1 \text{ lunar month} = 29.530589 \text{ days.}$$

In the course of a lunar month the Moon goes through a cycle of "phases" which are known as new, crescent, half, gibbous and full and then again in the reverse order, until it is new.

We have noted earlier that the Moon has a sidereal period of 27.3216615 days. The ecliptic is divided into 27 *nakṣatras.* Therefore, the Moon takes a little more than a civil day to cover the extent of a *nakṣatra* along the ecliptic. Starting from a new moon, the Moon gains about 12° per day over the Sun. The time taken by the Moon to cover exactly an amount of 12° relative to the Sun, is defined as a *tithi.* Thus, a lunar month has 30 tithis. The half-lunar month from a new moon to the succeeding full moon is called *śukla pakṣa* (bright fortnight) and the other half-lunar month from the full moon to the next new moon is a *kṛṣṇa pakṣa* (dark fortnight).

The lunar month is a natural unit for a month. It is also the interval between two successive full moons. It is important to note that the beginning and end of a lunar month are naturally marked by two successive new moons.

To summarize, we have the following special units of time which are defined by natural and observable occurrences with respect to the Sun and the Moon:

Mean civil day of 24 hours or 60 *ghaṭikās*

Sidereal day =		23h	56m	4.0s
Sidereal period of the Moon =	27d	7h	43m	11.6s
Lunar month =	29d	12h	44m	2.9s
Sidereal solar year =	65d	6h	9m	9.8s
Tropical solar year =	365d	5h	48m	45.3s

5.5 Luni-Solar Year
(OR LUNAR YEAR)

We saw in the earlier section that a lunar month is the period from one new moon to the next or from one full moon to the next. In our country, both the styles are followed. The lunar month reckoned from new moon to new moon is called *amānta* (or *mukhyamāna*). The one from full moon to another full moon is referred to as *pūrṇimānta* (or *gauṇamāna*). The lunar months are actually named after the corresponding solar months in which the initial new moon of the *amānta* lunar month occurs. A solar month is calculated from the entry into a *rāśi* by the Sun (*saṅkrānti*) to his entry into the next *rāśi*. For example, if a new moon falls in the solar month of *Caitra*, then the *amānta* lunar month commencing from this new moon is also called *Caitra*.

In the case of *pūrṇimānta* lunar calendar, a lunar month of this system commences from the moment of the full moon exactly a *pakṣa* (fortnight) earlier than the initial new moon, from which time the *amānta* month of the same name commences. For example, the *pūrṇimānta* month of *Caitra* commences a *pakṣa* earlier than the *amānta* month of *Caitra*. The *kṛṣṇa pakṣa* (dark half) of a lunar month is also called *vadi* and the *śukla pakṣa* (bright fortnight) is known as *sudi*.

The *amānta* calendar is followed for all purposes, namely, the counting of days for civil activities, festivals, religious functions, etc. in the states of Karnataka, Andhra Pradesh and Maharashtra. The lunar

(or luni-solar) year in this system commences with *Caitra śukla pratipat*, following *amāvāsyā*. In Gujarat also, an *amānta* calendar is followed but the lunar year starts after the Diwali new moon. That is, the Gujarati lunar year commences with *śukla pratipat* of the *Kārtika* month and hence it is called *Kārtikādi*. In Kutch and some parts of Saurashtra, the lunar year commences with the *śukla pratipat* of the *Āṣāḍha* month. In the states of Tripura, Assam, Bengal, Tamil Nadu and Kerala, the solar calendar is followed for dates and civil purposes, while the lunar calendar is used for festivals and religious purposes.

The *pūrṇimānta* lunar year is widely used in Bihar, Uttar Pradesh, Madhya Pradesh, Rajasthan, Haryana and Kashmir. But, in Orissa and Punjab though the solar calendar is generally followed, the lunar calendar used is *pūrṇimānta*.

An interesting feature of the *pūrṇimānta* calendar is that, although the first month of the lunar year, viz. *Caitra*, starts from the full moon, a *pakṣa* prior to the new moon (from which the *amānta Caitra* as well as the lunar year commences), the actual new year commences at the same time as the *amānta* new year. This means that the first half of the *pūrṇimānta Caitra*, which is *vadi* or *kṛṣṇa pakṣa* goes to the previous lunar year, while the second half of the *pūrṇimānta Caitra*, which is *sudi* or *śukla pakṣa* belongs to the new lunar year. Thus, the *pūrṇimānta* new lunar year starts from the middle of the *pūrṇimānta Caitra* month.

Table 5.1 Lunar months in a Luni-solar year

Sl. No.	Lunar month
1.	*Caitra*
2.	*Vaiśākha*
3.	*Jyeṣṭhā*
4.	*Āṣāḍhā*
5.	*Śrāvaṇa*
6.	*Bhādrapada*
7.	*Āśvina* (*Āśvayuja*)
8.	*Kārtika*
9.	*Agrahāyaṇa* (*Mārgaśirṣa*)
10.	*Pauṣa* (*Puṣya*)
11.	*Māgha*
12.	*Phālguṇa*

As pointed out earlier, the duration of a lunar month (mean) is 29.530589. How the lunar (or luni-solar) year is linked to the solar year, will be discussed in the next section.

5.6 *ADHIKAMĀSA* AND *KṢAYAMĀSA*

We saw that a lunar month, the interval between two successive new moons or full moons, is equal to 29.530589 days. Therefore, a lunar year, consisting of 12 such lunar months, has $12 \times 29.530589 = 354.36706$ days. But, the mean length of a (sidereal) solar year is 365.25636 days. The difference between a solar year and a lunar year, therefore, is obtained by

$$365.25636 - 354.36706 = 10.8893 \text{ days.}$$

The tropical solar year is a natural unit in the sense that the seasons are determined on the basis of the tropical solar year. However, in India, the sidereal solar year is used. A lunar month is also natural, in fact more so, since the lunations (new moon and full moon) can be directly observed. Therefore, it will be very useful and natural if the solar year is "coupled" with the lunar year (of 12 lunar months). This is done in a very systematic and natural way (unlike the leap-year of the Roman calendar).

In our country, both sidereal solar and lunar years are used for religious as well as civil purposes. We found that a lunar year falls short of a solar year by about 10.8893 days. When this difference adds up to a full lunar month, an extra month (called *adhikamāsa*) is added to that particular lunar year.

A normal lunar year consists of 12 lunar months and a duration of 354 or 355 days. The beginning of the lunar year is at the instant of the new moon (i.e., final moment of *amāvāsyā* of the previous lunar month) occurring in the course of the solar *Caitra* (i.e., when the Sun is in *Mīna rāśi*). This lunar month is also called *Caitra*. However, this is not the practice in Tamil Nadu. The second month of the lunar calendar, viz. *Vaiśākha,* starts at the following new moon and so on. In Fig. 5.1, N_0, N_1, N_2, etc., refer to new moons.

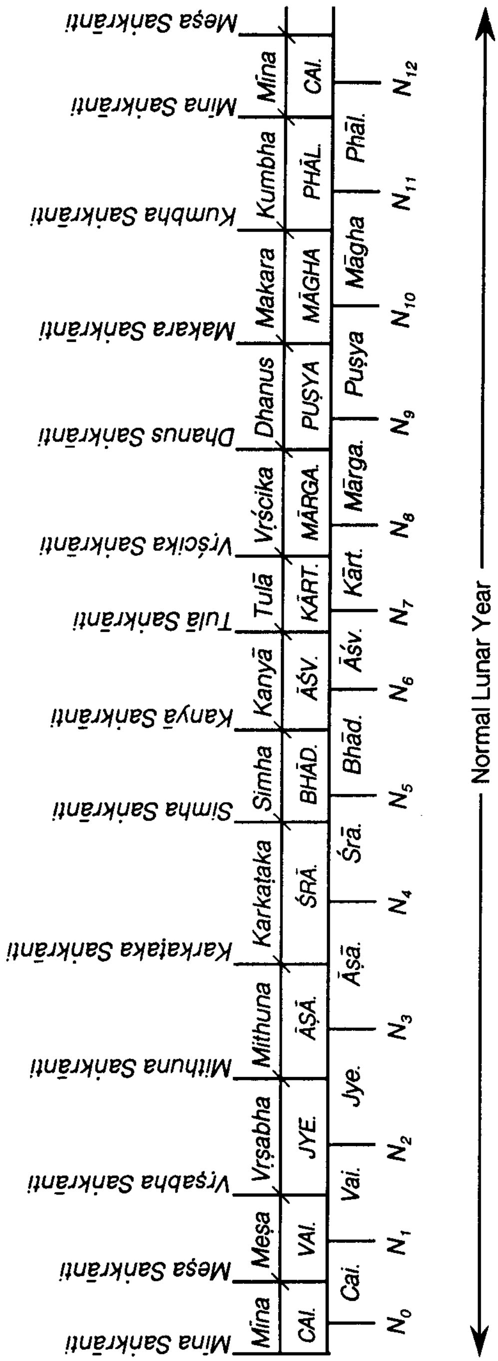

Figure 5.1 Normal lunar year of 12 months

In the case of a normal lunar year, in every (lunar) month there is a (sidereal) solar ingress (*saṅkramaṇa* or *saṅkrānti*) as shown in Fig. 5.1. In Tamil Nadu the names of the solar months are *Mīna* or *Phālguṇa*, *Meṣa* or *Caitra*, etc.

But, sometimes it may happen that a lunar month falls completely within the period of the corresponding solar month. In that case, during that lunar month, there is no *saṅkramaṇa* or entry of the Sun into the next *rāśi*. Such a lunar month is considered as a *adhikamāsa* (intercalary or extra month). During an *adhikamāsa*, usually, no Hindu religious ceremony is performed. A lunar year in which there is an *adhikamāsa* will have 13 months and 383 or 384 days. An example of the occurrence of an *adhikamāsa* is illustrated in Fig. 5.2. This refers to the *Śālivāhana śaka* year 1913 completed (i.e., the year 1991–92 AD).

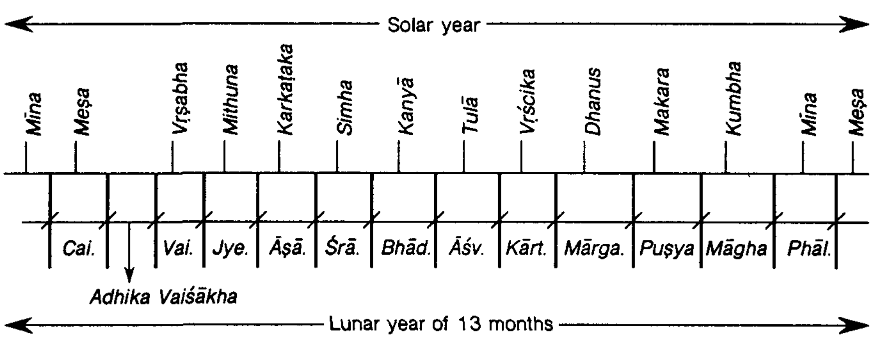

Figure 5.2 A lunar year of 13 months

An *adhikamāsa* occurs after 33 months, and a particular month becomes an *adhikamāsa* generally once in 19 years. It is found that the lunar month of *Māgha* never becomes an *adhikamāsa*. Generally, an *adhikamāsa* occurs between the months of *Phālguṇa* and *Āśvina*.

If, in the course of a lunar month, there are two *saṅkrāntis*, then that lunar month is considered as a *kṣayamāsa* (deletory month) which occurs rather rarely. In fact, generally a *kṣyamāsa* occurs once in 141 years, although once in a way it may occur after 19 years. A *kṣayamāsa* occurs in one of the three lunar months – *Kārtika*, *Mṛgaśira* or *Puṣya*. This is so, because there is a possibility of two *saṅkrāntis* in these particular lunar months due to the faster motion of the Sun during that period (the Sun will be around its perigee) and hence the solar months could be

very slightly less than the corresponding lunar month. A month which is a *kṣayamāsa* is not counted by its name at all. A lunar year in which there is a *kṣayamāsa* will have two *adhikamāsas*, one preceding it and another succeeding it.

REMARKS

We mentioned earlier that a *kṣayamāsa* occurs generally once in 141 years although it may occur, once in a way, in every 19 years. Bhāskara II in his **Siddhāntaśiromaṇi** mentions that a *kṣayamāsa* may occur in 19 or 122 or 141 years. The rationale appears to be as follows.

According to Bhāskara, 5311 *adhikamāsas* occur in 14,400 solar years. Now, converting 14400 / 5311 into a continued fraction, we get

$$\frac{14400}{5311} = 2\ \frac{1}{+1}\ \frac{1}{+2}\ \frac{1}{+2}\ \frac{1}{+6}\ \frac{1}{+1}\ \frac{1}{+1}\ \frac{1}{+7}\ \frac{1}{+8}\ \frac{1}{+2}$$

The successive "convergents" of this continued fraction are

2/1, 3/1, 8/3, 19/7, 122/55, 141/62.

The interval between a new moon and the next *saṅkrānti* (solar ingress into a *rāśi*) is called a *śuddhi*. It is the interval gained by the *cāndramāna* over the *sauramāna*. The significance of the convergents 19/7, 122/55 and 141/62 is that in 19 years, 122 years and 141 years there are respectively 7,55 and 62 *adhikamāsas*. Therefore, if there is a *śuddhi* of a certain number of days in a particular year, the same *śuddhi* will repeat in durations of 19 years, 122 years and 141 years. This is responsible for the occurrence of a *kṣayamāsa* once in 19,122 and 141 years.

SEASONS

Actually, the importance of the tropical solar year lies in its being naturally related to seasons which repeat periodically once a year.

We mark the beginning of a season by the position of the Sun. In the temperate zone, the *spring* season begins when the Sun crosses the celestial equator going northward and crossing the vernal equinox ♈ (March 22). The second season, *summer*, begins when the Sun reaches its northernmost point in its annual journey, called the *summer solstice* (June 22). The next season, *autumn*, begins when the Sun crosses the

celestial equator again, in its southward course at the autumnal equinox ♎ (Sept. 23). The fourth and the last season, *winter,* commences when the Sun is at the *winter solstice* (Dec. 22).

During the spring season, from March 22 to June 22, after the end of the earlier cold period, there is more sunshine and light; days grow longer and trees bloom.

From June 22 to September 23, during the summer, the days become hotter. This is a season of agricultural activity.

Then the days become gradually shorter and cooler and the trees shed their leaves during the autumn (Sept. 23 to Dec. 22).

Finally, in winter (Dec. 22 to Mar. 22), the days are short and cold.

The above description of the seasons and their periods in the course of a (tropical) solar year refers to the Earth's northern hemisphere. On the other hand, for places in the southern hemisphere, the seasons and their effects will be in the reverse order. For example, when it is summer in the northern hemisphere, it will be winter in the southern hemisphere.

However, this division of seasons is not suitable for India which has a tropical climate. The Indian system recognises three main seasons of hot, rainy and cold which are further subdivided.

The *ṛtus* (seasons) start with *śiśira* at the winter solstice and cover two months each. The six *ṛtus* are as follows:

	Season	**Description**	**Period (approximately)**
1.	*Vasanta*	Spring	Feb. 19 to Apr. 19
2.	*Grīṣma*	Summer	Apr. 20 to June 20
3.	*Varṣa*	Rainy	June 21 to Aug. 22
4.	*Śarat*	Autumn	Aug. 23 to Oct. 23
5.	*Hemanta*	Dewy or snowy	Oct. 24 to Dec. 21
6.	*Śiśira*	Cold	Dec. 22 to Feb. 18

However, as noted earlier, it is the *tropical* year, and not the sidereal, which determines seasons.

The dates of the beginning and end of each season given are approximate and valid for the current period (say about a century). Due to the precession of the equinoxes, these dates gradually change over a long period. However, if the sidereal solar months are considered, which is

not correct, then the commencement and end of the seasons are fixed with reference to these months.

For example, the **Taittirīya Samhitā** gives the names of the solar months connected with the different seasons as follows:

	Season	Solar Months (Vedic)
1.	*Vasanta*	*Madhu* *Mādhava*
2.	*Grīṣma*	*Śukra* *Śuci*
3.	*Varṣa*	*Nabhas* *Nabhasya*
5.	*Hemanta*	*Sahas* *Sahasya*
4.	*Śarat*	*Iṣa* *Ūrja*
6.	*Śiśira*	*Tapas* *Tapasya*

5.8 *Yuga* System

As mentioned at the beginning of this chapter, on the macrocosmic scale of time, the *yuga* system has evolved in Indian astronomy. Many important elements of the planets and other parameters are given in terms of the number of revolutions in the course of a long period of time called *yuga*.

While in the **Vedāṅgajyotiṣa** the word *yuga* was used to mean a period of 5 years, in later works the word meant a large period of time. So far as Indian astronomy is concerned, the *yugas* of large period of time have been used to indicate the rate of motion of the planets and other important points of astronomical significance. This technique enabled them to express these constants as integers, though very large, thus avoiding the inconvenient fractions.

One *Mahāyuga* of 43,20,000 years comprises four *yugas* (or *yugapādas*), viz., *Kṛta, Tretā, Dvāpara* and *Kali*. Āryabhaṭa took them

all to be of equal duration, 10,80,000 years each. But all other astronomers, except Lalla and Vateśvara, have adopted the system of four *yugas* which have their durations in the ratio of 4:3:2:1. Thus,

Kṛtayuga :	17,28,000 years
Tretāyuga :	12,96,000 years
Dvāparayuga :	8,64,000 years
Kaliyuga :	4,32,000 years
Total period of a *Mahāyuga* :	43,20,000 years

Note that a *Mahāyuga* is ten times a *Kaliyuga.*

1 *Manvantara* = 71 *Mahāyugas* = 30,67,20,000 years
14 *Manvantaras* = $14 \times 30672 \times 10^4 = 4.294 \times 10^9$ years

Between two successive *manvantaras*, there is a *sandhyā* period equal to the duration of a *Kṛtayuga*. For the fourteen *manvantaras,* there are 15 *sandhyās* (including one before the first *manvantara* and one after the last).

A *kalpa* is formed by the fourteen *manvantaras* along with their *sandhyas,* so that

$$1\ kalpa = (14 \times 30{,}67{,}20{,}000) + (15 \times 17{,}28{,}000)$$
$$= 432 \times 10^7 \text{ years.}$$

Thus, a *kalpa* is one thousand times a *Mahāyuga.*

At present, we are under what is called *Śvetavarāha kalpa* in which six *manvantaras* have already elapsed and we are now in the seventh one called *Vaivasvata manvantara*. In this *manvantara*, 27 *Mahāyugas* have elapsed and we are now in the 28th *Mahāyuga*. Again, in this running *Mahāyuga*, the first three *yugas,* viz., *Kṛta, Tretā* and *Dvāpara* are over. The fourth, *Kaliyuga*, is currently running. Astronomers are agreed on the point that the present *Kaliyuga* commenced at midnight between the 17th and 18th February 3102 BC (by the Julian reckoning). For the year 1992 AD, 5093 years have elapsed and the actual number of days (called *kali ahargaṇa*) as on the midnight of January 1–2, 1992 is 18,60,158.

The Indian astronomical *siddhāntas* assumed that at the commencement of the *kalpa*, all the planets including *Ketu* were in

conjunction (i.e., at the same celestial longitude) at the first point of Aries, and the ascending node (*Rāhu*) of the Moon was 180° away, i.e., at the first point of Libra.

As pointed out earlier, the *yuga* theory has been adopted by Indian astronomers to express the mean motions of planets and other important geometrical points fairly accurately. These details will be discussed in a subsequent chapter.

5.9 Indian Eras

An "era" is important in preparing calendars which are used for civil purposes as well as official, religious, historical and chronological records and events.

Among several different eras in use, the most popular ones are the *Kali, Vikrama śaka, Śālivāhana śaka* and *Kollam*, besides the *Hejira* era used by Muslims. We will describe these important eras one by one. In any era that is adopted, there will be a starting point, called an *epoch*, from which day, for future periods, years, months and days are counted. For example in the Christian era, which we have been following currently for civil purposes, years and dates are counted from a reference day about 1999 years ago. The reference day of an era might be an historical event like the coronation of a famous king or the birth of a great personality or an astronomical event of significance.

1. *KALI ERA*

In Indian astronomical texts, generally, this era is adopted. As pointed out earlier, the *Kaliyuga* is supposed to have commenced from the midnight between 17th and 18th of February, 3102 BC (Julian), the day following the midnight being a Friday. Actually, an astronomical reference to the *Kali* era is available in the famous text **Āryabhaṭiyam** composed by the renowned astronomer and mathematician, Āryabhaṭa I (476 AD). He says that he was 23 years old when 3600 years had elapsed in the *Kaliyuga*. That year corresponds to 499 AD, when the celebrated astronomer composed his immortal text. It has been inferred that this era was used long before Āryabhaṭa I, although earlier records to that effect are yet to be discovered.

The *Kali* era has an advantage over later eras for the simple reason

that it covers the antiquity of our Indian civilization adequately which other eras cannot, since those were started later. However, for the recording of contemporary events many others eras were adopted.

The beginning of *Kaliyuga* is characterized by the end of the *Mahābhārata* war (Āryabhaṭa refers to the end of *Dvāparayuga* as *Bhāratāt pūrvam*) and also the demise of Śrī Kṛṣṇa, according to the *Purāṇas.*

2. *VIKRAMA ŚAKA*

This era is widely used in most states of north-west India which follow the *pūrṇimānta* lunar calendar (i.e, lunar month ranging from one full moon to the succeeding full moon). This era is used in Gujarat also, although the *amānta* (new moon to new moon) lunar month is followed, but the lunar year starting from the Diwali new moon is also taken into account. The starting epoch of the *Vikrama śaka* is 58 BC, but the source of its origin is not precisely known. The popular belief is that the era was started by King Vikramāditya of Ujjayinī to commemorate his victory over the Śakas or Scythians; but inscriptional or other evidence for this is not available. Actually this era was associated with the Malavas and, hence, it was known as the Malava era for a very long time, from the early 5th century AD. Earlier it was also called the *Kṛta* era, but we do not know why it was called so. It is also argued that this era was started by the Gupta Emperor, Candragupta II, who defeated the Śakas. But we are yet to arrive at a consensus on many issues of importance concerning Indian chronology.

3. *ŚĀLIVĀHANA ŚAKA*

This era starts from 78 AD and is very widely used for both solar and lunar calendars. Like the *Vikrama* era, the origin of the *Śaka* era is not well understood. One theory is that the Kusana emperor, Kanishka, the king who came after Asoka, started this era from the date of his accession. It is opined by some that Kanishka used the old *Śaka* era omitting 200 years; this opinion presumes that there was a *Śaka* era 200 years earlier. However, the date of Kanishka's reign is quite uncertain. And, in fact, no Indian text makes any reference to support this theory.

4. *KOLLAM ERA*

This era is in use in Kerala and its origin is again not clear. This era is sometimes referred to as the *Paraśurāma* era and the belief is that the Kollam era came into practice by omitting thousands of years from the earlier extant *Paraśurāma* era. The year in the Kollam era starts from the entry of the Sun into *Simha rāśi*; the era started from 824 AD.

5. *HEJIRA ERA*

The *Hejira* era is followed by the Muslims. The era starts from 622 AD and is based on lunar calculations. The year begins with *Muharram*.

This era of the Mohammedan calendar, probably introduced by the Caliph Umar about 638–39 AD, started from the evening of July 15, 622 AD, Thursday (giving rise to Friday, commencing that evening) when the crescent Moon of *Muharram* was first visible. This was the new year day preceding the emigration of Prophet Muhammad from Mecca.

Many other eras have been in use, like *Saptarṣi, Yudhiṣṭhira, Buddha nirvāṇa, Mahāvīra nirvāṇa, Bengali San, Lakṣmaṇasenā, Tarikh Ilahi* (starting from 1555 AD, introduced by King Akbar), etc.

5.10 Time on a Microcosmic Scale

In the course of a day, to fix actual *muhūrta* (auspicious time) for rites and rituals and marriages and also for routine civil activities, smaller units of time are needed. In Indian astronomy, according to the **Sūryasiddhānta**, the measurement of time on the micro-scale starts with *prāṇa* (or *asu*). The length of a *prāṇa* (inhalation or exhalation) is defined as the time taken for pronouncing 10 long syllables (*gurvakṣara*). It is also the time taken to inhale by a person in normal health. The table of time-units used in the course of a day is given below:

10 *guruakṣara*	=	1 *prāṇa* (*asu*, about 4 seconds)
6 *prāṇas*	=	1 *vināḍī*
60 *vināḍī*	=	1 *nāḍī*
60 *nāḍīs*	=	1 *day*

A *nāḍī* is also referred to as a *nāḍikā* or a *ghaṭikā*.

There are also other units of time. In the *Purāṇic* tradition, we have

15 *nimeṣas*	=	1 *kāṣṭhā*
30 *kāṣṭhās*	=	1 *kalā*
30 *kalās*	=	1 *muhūrta*
30 *muhūrtas*	=	1 day

Here, one *nimeṣa* means 'in the twinkling of an eye'.

Bhāskara II (1114 AD), in his **Siddhāntaśiromaṇi,** provides another table:

100 *truṭi*	=	1 *tatpara*
30 *tatparas*	=	1 *nimeṣa*
18 *nimeṣas*	=	1 *kāṣṭhā*
30 *kāṣṭhās*	=	1 *kāla*
30 *kālas*	=	1 *ghaṭikā*
2 *ghaṭikās*	=	1 *kṣaṇa*
30 *kṣaṇas*	=	1 *dina*

It is interesting to note that the smallest unit of time *truṭi,* given by Bhāskara II, is 2916000000th of a day. In terms of our modern unit of time, viz., seconds, it is

$$1 \textit{ truṭi} = \frac{1}{33750} \text{ second.}$$

Furthermore, Bhāskara II explicitly mentioned that the day taken for division into 60 *ghaṭikās* (or *nāḍikās*) is the sidereal day (*nākṣatra dina*) which is of the duration of 23h, 56m, 4s and not 24 hours. Thus, a *ghaṭikā* (or *nāḍikā*) is a little less than 24 minutes.

6

Calendars and the Indian *Pañcāṅga*

6.1 Introduction

A certain "calendar" system is necessary to keep a record of day-to-day activities as well as special events and natural occurrences. Broadly speaking, a calendar is used to identify a particular *day* as belonging to a particular *month* of a particular *year*. While the particular day is identified with a *date* of the said month and year, the calendar should specify the *weekday* on which the given date falls. Obviously, there ought to be no confusions and contradictions in a well thought out calendar system. In different societies, depending on their requirements and practices – religious, social and civil as also on the levels of their computational accomplishments – different calendar systems have been evolved. These calendar systems are essentially based on the solar year, either tropical or sidereal, or lunar (or luni-solar) year and lunations, i.e., the new moon and the full moon.

The basis of the Roman calendar – also popularly referred to as the Christian calendar – is the tropical solar year. As pointed out earlier, for an observer on the Earth, the time taken by the Sun to complete a revolution along the ecliptic with reference to the vernal equinox (the first point of Aries) is one *tropical year* (*sāyana saura varṣa*). Its average duration is 365.24219 days. But, for civil use it is convenient to have a whole number of days in a year and hence a civil year is considered normally as consisting of 365 days. Now, in order to account for the residual part, 0.24219^{th} day, Julius Caesar added one extra day once in four years. That year of the extra day is called a *leap year* consisting of 366 days.

Since a year has 12 months, the total number of days of the year are distributed among them. Some months have 30 days and the others 31 days. Julius Caesar is said to have a particular month privileged with

31 days and named July in his own honour. However, his successor, Emperor Augustus, named the month succeeding July as August, after himself, and endowed it with 31 days again. Due to this special arrangement of having 31 days in two successive months, February is impoverished and has only 28 days in the ordinary years and 29 days in leap years. This calendar is referred to as the *Julian Calendar.*

6.2 Gregorian Calendar

With the introduction of the Julian Calendar, the difference between a civil year and the natural tropical year was reduced to a great extent. But still, it amounted to an excess of 0.00781th of a day (i.e., 11 minutes, 15 seconds) over the tropical year. In the course of a 100 years, this difference accumulates to 0.781th of a day.

The religious head of the Roman Catholic Church as that time, Pope Gregory XIII (1572–1585 AD), introduced a further change in the Roman Calendar to compensate for this excess. By the latter part of the 16th century, the excess in the civil year had accumulated to about 10 days.

The following changes were introduced in the new style (N.S.) calendar, now popularly called the Gregorian Calendar, with effect from 1582 AD:

1) The day succeeding October 4, 1582, Thursday, would be considered as October 15, 1582, Friday. In other words, October 5 was changed to October 15, thus shedding the extra 10 days.
2) As in the Julian Calendar, there would be a leap year once in four years; in the leap year, an extra day is added to February. Ordinarily, if a year of the Christian era is divisible by 4, then that year is a *leap-year* with some exceptions.
3) The century years, viz., 1600,1700, etc., will be leap-years only if they are divisible by 400 (and not just by 4). Thus, while 1600 AD and 2000 AD are leap-years, the century years, 1700, 1800 and 1900 AD are not leap-years.

After the introduction of the Gregorian Calendar, the difference between a civil year and a tropical solar year was reduced to just about 0.1216th of a day (i.e., 2 hours, 55 minutes, 6 seconds) in the course

of 400 years. This small difference adds up to a full day in the course of about 3300 years. After that many years, a day will have to be dropped from the civil calendar.

6.3 Hindu Calendar

The Indian calendar has evolved over thousands of years since the Vedic times. In the course of the developments of Indian astronomy through its different phases and periods – Vedic, **Vedāṅgajyotiṣa** and Siddhāntic – the Hindu calendar has continuously undergone refinements. There are essentially two systems followed in the Indian calendar, viz. *luni-solar* and purely *solar*. We shall briefly consider these two systems that are in use.

(i) LUNI-SOLAR CALENDAR

As pointed out earlier, the *amānta* system, in which the lunar month commences with an *amāvāsyā* and ends with the succeeding one, is generally followed in Karnataka, Andhra Pradesh, Maharashtra and Gujarat. On the other hand, in the *pūrṇimānta* system, the lunar month begins with one full moon and ends with the next full moon. This system is followed in most of the north Indian states.

In the *amānta* system, the luni-solar (or lunar) year starts with *Caitra śukla pratipat* and ends with *Phālguṇa kṛṣṇa amāvāsyā.* On the other hand, in the *pūrṇimānta* system, while lunar year again starts with *Caitra śukla pratipat* as in the case of the *amānta* system, the first lunar month *Caitra* itself will have commenced a fortnight earlier. In other words, the first half of the *pūrṇimānta Caitra* coincides with the second half of *Phālguṇa* of the *amānta* system.

Each lunar year is called a *samvatsara.* A cycle of 60 *samvatsaras* is followed. This cycle is five times the Jovian cycle (or *Bārhaspatya* cycle) of 12 years. The names of the sixty *samvatsaras* are given in the Table 6.1.

The lunar new year commences with the *Cāndramāna Yugādi*, the first day of the (*amānta*) *Caitra* which follows the new moon just preceding the *Meṣa saṅkramaṇa* (i.e., before about April 14). For example, in 1993, the *Cāndramāna Yugādi* falls on March 24, Wednesday, when the lunar new year commences. This first day of

Table 6.1 Names of *Samvatsaras*

No.	*Samvatsara*	No.	*Samvatsara*	No.	*Samvatsara*
1.	*Prabhava*	21.	*Sarvajit*	41.	*Plavaṅga*
2.	*Vibhava*	22.	*Sarvadhārin*	42.	*Kīlaka*
3.	*Śukla*	23.	*Virodhin*	43.	*Saumya*
4.	*Pramoda*	24.	*Vikṛta*	44.	*Sādhāraṇa*
5.	*Prajāpati*	25.	*Khara*	45.	*Virodhakṛt*
6.	*Aṅgīrasa*	26.	*Nandana*	46.	*Paridhāvin*
7.	*Śrīmukha*	27.	*Vijaya*	47.	*Pramādin*
8.	*Bhāva*	28.	*Jaya*	48.	*Ānanda*
9.	*Yuvan*	29.	*Manmatha*	49.	*Rākṣasa*
10.	*Dhātri*	30.	*Durmukha*	50.	*Anala (Nala)*
11.	*Īśvara*	31.	*Hemalamba*	51.	*Piṅgala*
12.	*Bahudhānya*	32.	*Vilamba*	52.	*Kālayukta*
13.	*Pramāthin*	33.	*Vikārin*	53.	*Siddhārthin*
14.	*Vikrama*	34.	*Śārvari*	54.	*Raudra*
15.	*Vṛṣa*	35.	*Plava*	55.	*Durmati*
16.	*Citrabhānu*	36.	*Śubhakṛt*	56.	*Dundubhi*
17.	*Subhānu*	37.	*Śobhana*	57.	*Rudhirodgārin*
18.	*Tāraṇa*	38.	*Krodhin*	58.	*Raktākṣi*
19.	*Pārthiva*	39.	*Viśvavasu*	59.	*Krodhana*
20.	*Vyaya*	40.	*Parābhava*	60.	*Kṣaya (Akṣaya)*

the *Caitra* month immediately follows the new moon preceding the *Meṣa Saṅkrānti* which falls on April 13, 1993. In fact, this lunar year (1993–94) corresponds to the *Śrīmukha samvatsara* and *Śālivāhana śaka* year 1915 elapsed. In the *Kali* era, the new lunar year corresponds to the year 5094 elapsed.

We can use the following conversion formulae to obtain the corresponding equivalents to a given year of the (Christian) Gregorian calendar. If Y is the Gregorian year in which the (*amānta*) lunar *Caitra* falls, then

(i) The corresponding *Kali* year (elapsed) is obtained by

$$Kali = Y + 3101$$

eg. for $Y = 1993$ (i.e., lunar year 1993–94).

$$Kali = 1993 + 3101 = 5094 \text{ (elapsed)}$$

(ii) The *Śālivāhana śaka* year is obtained by

Śaka = Y–78

eg. for Y = 1993, *Śaka*: 1993–78 = 1915 (elapsed).

(iii) The *samvatsara* number, *Sam*, is obtained by

Sam = Remainder after dividing (Y–1926) by 60.

Then, the name of the *samvatsara* is obtained from Table 6.1 corresponding to the number *Sam*, thus obtained,
e.g. For Y = 1993, we have

Sam = Remainder after dividing (1993–1926) by 60

i.e., 67 by 60 which is 7. From Table 6.1, for *Sam* = 7, we get the name of the *samvatsara* as *Śrīmukha*.

Example: Find the *Kali* year, *Śaka* year and the *samvatsara* for the year 1944 (i.e., lunar year 1944–45).
We have Y = 1944. Therefore,

(i) *Kali* year = Y+3101 = 1944+3101 = 5045 (elapsed)
(ii) *Śaka* year = Y–78 = 1944–78 = 1866 (elapsed)
(iii) Y–1926 = 1944 – 1926 = 18 = *Sam*.

From Table 6.1, *Sam* = 18 corresponds to *Tāraṇa*.

Note: The Gregorian year starts on January 1 while the Indian year starts in March/April. This fact has to be taken into consideration while determining the years elapsed in any of the Indian eras.

(ii) SOLAR CALENDAR

Among the Hindus a purely solar calendar is also used. The solar (sidereal) year is the time taken by the Sun to go round the ecliptic once with reference to the fixed stars. The solar year starts when the Sun enters the constellation *Meṣa*, i.e., with the *Meṣa saṅkramaṇa* (or solar ingress into *Meṣa*). In the current period it is around April 14 (or 13 or 15). The solar year is divided into 12 solar months. The length of any particular solar month is the duration of the stay of the Sun in that particular *rāśi* (of 30° extent) of the zodiac. Some solar months are longer and some shorter, since the Sun moves faster near its perigee and slower near the apogee.

The solar months are named after the *rāśis* the Sun occupies during these months, as *Meṣa, Vṛṣabha,* etc. But, more popularly, the names of the solar months are the same as those of the lunar months, viz., *Caitra, Vaiśākha,* etc. These names are prefixed as "*saura*" to distinguish them from the lunar. The solar year commences with *saura Vaiśākha* around April 14 and ends with *saura Caitra.* The names of the solar months are ambiguous, followed differently in the different regions of India. Therefore, it is better to name them after *rāśis* occupied by the Sun.

(iii) HINDU FESTIVALS

Most of the Hindu festivals are based on the luni-solar (or lunar) calendar. Each of these falls on a particular *tithi* of a specified *pakṣa* in a particular lunar month. The lunar new year's day – called *Cāndramāna Yugādi* – falls on *śukla pratipat* of the (*amānta*) *Caitra* month. ŚRĪ RĀMANAVAMĪ is celebrated eight days later, i.e., on *śukla navamī* of the same *Caitra* month. Similarly, ŚRĪ KṚṢṆA JANMĀṢṬAMĪ falls on *aṣṭamī* of the *kṛṣṇa pakṣa* in the month of (*amānta*) lunar *Śrāvaṇa.*

GANEŚA CATURTHĪ when Lord Ganeśa or Vināyaka is worshipped coincides with *śukla caturthī* of the *Bhādrapada* month. MAHĀLAYA AMĀVĀSYĀ falls on (*amānta*) *Bhādrapada amāvāsyā,* i.e., on the last of the month in which *Ganeśa caturthī* falls. The ten days following the *Mahālaya amāvāsyā* are celebrated as DASARA (or "DUSSEHRA") from *Āśvayuja śukla pratipat* to *daśamī.* In Bengal this period is celebrated as DURGĀ PŪJĀ. The DASARA procession of Mysore celebrating the slaying of *Mahiṣāsura* by the goddess *Cāmuṇḍeśvarī* is very famous.

The tenth day of the DASARA period is celebrated as VIJAYADAŚAMĪ. In North India it is observed as RĀMLĪLĀ to celebrate *Śrī Rāma*'s killing *Rāvaṇa.* On this occasion, huge figures of *Rāvaṇa, Kumbhakarṇa* and *Meghanāda* are erected in public places and then set on fire using fireworks—to symbolize the destruction of evil and the triumph of the good.

DIWALI (or DĪPĀVALĪ) is a major festival celebrated throughout the country. Generally this festival is held for two days—*Naraka caturdasī* and *Dīpāvali* (*amāvāsyā*) on the following day—which fall on (*amānta*) *Āśvayuja kṛṣṇa caturdasī* and *amāvāsyā.* DIWALI is popularly known as the festival of lights.

MAHĀŚIVARĀTRĪ is an important day observed by Saivites, which falls on *amānta Māgha kṛṣṇa caturdasī.*

HOLI (or HOLIKĀ DAHANA) festival is held on *Phālguṇa pūrṇimā*, a fortnight prior to the YUGĀDI of the next lunar year. HOLI is a popular festival in North India; it is a celebration of colours.

Besides these major festivals, there are many others which are also based on the lunar calendar.

But there are also a few Hindu festivals which are based purely on the (sidereal) solar calendar. The beginning of the solar new year—the first day of the *saura Vaiśākha*—is celebrated as the New Year's day in Assam, Bengal, Orissa, Tamil Nadu and Kerala. This is the day that the Sun enters into *Meṣa rāśi*, and therefore the month is also called *Meṣa māsa*. However, in Tamil Nadu and Kerala the first solar month is called *Caitra*.

The beginning of the tenth month of the solar year, namely *saura Māgha*, is celebrated as *Makara Saṅkrānti* throughout the country. This festival is popular in Tamil Nadu and Kerala as PONGAL, and as MĀGHA BIHU in Assam.

There are also a few festivals which depend on the Moon's position relative to the stars. OṆAM is celebrated in Kerala when the Moon is in *Śravaṇa nakṣatra* in *saura Bhādrapada*. These days, usually, the solar *Bhādrapada* month runs from August 17 to September 16. During this period, the day when the Moon is in *Śravaṇa nakṣatra*, is observed as OṆAM. For example, in 1993, *Oṇam* falls on August 30, Monday. *Oṇam* is referred to as TIRU OṆAM also. In Kerala, *Oṇam* is a harvest festival celebrated with flowers and graceful dances by the womenfolk.

Among a section of the Hindus, Śrī Kṛṣṇa's birthday is celebrated when the Moon is in *Rohiṇī nakṣatra* in the *saura Bhādrapada* month. As noted earlier, the *saura Bhādrapada* extends from August 17 to September 16. During this period, ŚRĪ KṚṢṆA JAYANTĪ is celebrated on the day of *Rohiṇī nakṣatra*. For example, this day falls on September 8, Wednesday, in 1993. Generally, it occurs on *amānta Śravaṇ Kṛṣṇa aṣṭamī* or equivalently on *paurṇimānta Bhādrapada Kṛṣṇa aṣṭamī*.

6.4 ISLAMIC CALENDAR

The Islamic (or Mohammedan) calendar is purely lunar and is not related to the solar calendar. The year consists of 12 lunar months. The beginning of each month is determined by the

observation of the *crescent* moon in the evening sky. Therefore, in an Islamic year of 354 (or 355) days, each month has 29 or 30 days.

As pointed out earlier, the era of the Islamic calendar viz., the *Hejira* (A.H.) started from the evening of July 15, 622 AD when the crescent moon (following the new moon) of the first month, *Muharram*, was first visible. This was the new year day before the emigration of Prophet Muhammad from Mecca which was on September 20, 622 AD.

The twelve lunar months of the Islamic calendar and their fixed number of days are listed below:

1.	*Muharram*	30	7.	*Rajab*	30
2.	*Safar*	29	8.	*Shaban*	29
3.	*Rabi-ulawwal*	30	9.	*Ramadan*	30
4.	*Rabi-ussani*	29	10.	*Shawal*	29
5.	*Jamada' lawwal*	30	11.	*Zilkada*	30
6.	*Jamad-ussani*	29	12.	*Zilhijja*	29 (or 30)

The leap-year, in which the last month *Zilhijja* has one day more (i.e., 30 days), contains 355 days and is known as *Kabishah*. In a cycle of 30 years, there are 19 common years of 354 days and 11 leap-years of 355 days.

The rule for determining a leap-year is as follows: if after dividing the *Hejira* year by 30, the remainder is 2,5,7,10,13,16,18,21,24,26 or 29, then it is a leap-year.

6.5 The Indian Calendar and *Pañcāṅga*

In India, for a long time, both solar and lunar calendars have been used for calendrical purposes. We have already seen that the lunar month is a natural unit of time and that it is the interval between two successive new moons (or full moons). A lunar year consisting of about 354 days is pegged on to the solar year of about 365 days, by introducing the *adhikamāsas* (intercalary months) in a natural way based on the absence of a *saṅkrānti* in a lunar month.

Then a lunar month, during which there are waxing and waning phases of the Moon, is divided into two equal halves called *śukla pakṣa* and *kṛṣṇa pakṣa*. During the *śukla pakṣa* (bright half), the phases of the

Moon increase from new to full through the crescent, half and gibbous phases. Then, in the *kṛṣṇa pakṣa* (dark half), the Moon wanes from full to new in the reverse order. In a *pakṣa*, there are 15 days. Each *vāra* (week) has 7 days which are named as *Ravivāra, Somavāra,* etc. after the planets. The year in a calendar is identified by the number of years elapsed since the beginning of the ongoing era like *Kali* or *Śaka*.

6.6 What is the *Pañcāṅga*?

In a traditional Hindu household the annual *pañcāṅga* is indispensable. A Hindu uses the *pañcāṅga* for all his religious observances and also for ascertaining the dates of important religious festivals like GANEŚA CATURTHĪ, ŚRĪ RĀMANAVAMĪ, ŚRĪ KṚṢṆA AṢṬAMĪ, YUGĀDI as well as special days like *ekadasi, dvadasi, amāvāsyā* and *pūrṇimā*.

As the word indicates, the *pañcāṅga* consists of five parts (*pañcāṅga*), viz.,

(i) *Tithi*
(ii) *Nakṣatra*
(iii) *Vāra*
(iv) *Yoga* and
(v) *Karaṇa*.

In addition to the details about the above five parts, the *pañcāṅga* also contains a good deal of information which is of relevance in astrological, religious, social fields and also for predicting planetary positions, sunrise and sunset timings.

6.7 *Tithi*

In the course of a lunar month, from a new moon to the next new moon, the shape and size of the Moon changes from day to day. On an *amāvāsyā* day (new moon day) the Moon is invisible as in *A*, (see Fig.6.1). On the next day, a very thin "crescent" moon *B* is visible, if the sky is clear, soon after the sunset in the western horizon.

On the succeeding days of the *śukla pakṣa* the brighter or visible part of the Moon keeps on growing until it is half (*C*) on the 7th or

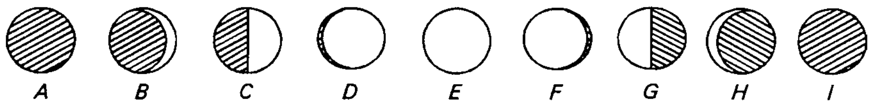

Figure 6.1 Phases of the Moon

8th day after the new moon. Also, each day, the Moon keeps moving up in the sky at sunset since it moves farther from the Sun at the rate of about 12° per day. When the phase of the Moon is half, it will be midway in the sky between the eastern and the western horizons. Then, each succeeding day the brighter part of the Moon grows more than half when it is said to be gibbous, *D*. At the end of the *śukla pakṣa*, the Moon will be fully visible, E, when the *kṛṣṇa pakṣa* (dark half of the month) commences.

In the *kṛṣṇa pakṣa*, the phases of the Moon diminish (or wane) in the reverse order. From the full moon upto the 7th or 8th day more than half the Moon is bright when it is said to be gibbous, *F*. Then, on *saptami* or *aṣṭamī*, the Moon will be half(*G*). The bright portion of the Moon goes on decreasing till it is crescent (*H*) again, a day before the new moon day. At the end of the *kṛṣṇa pakṣa* the Moon is totally invisible on the new moon day, *I*.

The lunar month is divided into 30 parts called *tithis*. The bright half has 15 *tithis* and so too the dark half. The duration of a *tithi* is the time taken by the Moon to move 12° relative to the Sun. The durations of different *tithis* are not equal. In a *pakṣa* (fortnight), starting from the new moon or the full moon, there are 15 *tithis*:

1. *Pratipat*
2. *Dvitiya*
3. *Tritiya*
4. *Caturthī*
5. *Panchami*
6. *Shashti*
7. *Saptami*
8. *Aṣṭamī*
9. *Navamī*
10. *Daśamī*
11. *Ekadasi*
12. *Dvadasi*

13. *Trayodasi*
14. *Caturdasi*
15. *Pūrṇimā* or *Amāvāsyā*

Therefore,

$$Tithi = [\text{Longitude of the Moon} - \text{Longitude of the Sun}] \,/\, 12^\circ$$

where both the longitudes are in degrees.

The quotient part indicates the number of *tithis* completed during the lunar month, and hence the quotient+1 gives the currently running *tithi*. If the *tithi* is less than 15, then it is of the *śukla pakṣa* (bright fortnight). If the *tithi* is greater than 15, then subtract 15 from that number and the remainder gives the running *tithi* in the *kṛṣṇa pakṣa*.

If the *tithi* is 15, then it is a *pūrṇimā* (full moon) day and if it is 30, the day is an *amāvāsyā* (new moon day).

Note: If the longitude of the Moon is less than that of the Sun then add 360° to avoid the negative sign, and then divide it by 12 to get the *tithi*.

Example: On March 21, 1990, at 5h 30m a.m. (IST), the longitude of the Sun is 11s 6° 23′ 13″ and the longitude of the Moon is 8s 22°10′ (according to the *Rāṣṭrīya Pañcāṅga*, published by the Govt. of India).

The Moon's longitude is 8s 22° 10′ which means that the Moon has covered 8 signs (*rāśis*) i.e., $8 \times 30^\circ = 240^\circ$ and then has traversed 22°10′, so that the longitude is

$$\begin{aligned} M &= 240^\circ + 22^\circ 10' = 262^\circ 10' \text{ in degrees} \\ &= 262.16666^\circ \text{ (in the decimal notation).} \end{aligned}$$

Similarly, the longitude of the Sun is

$$\begin{aligned} S &= (11 \times 30^\circ) + 6^\circ 23' 13'' = 336^\circ\, 23'\, 13'' \\ &= 336.38694^\circ. \end{aligned}$$

Since M is less than S, add 360° to (M–S) and then divide it by 12° to get the *tithi*, *T*. Therefore,

$$\begin{aligned} T &= (M - S + 360^\circ / 12^\circ \\ &= (262.16666^\circ - 336.38694^\circ + 360^\circ) / 12^\circ \\ &= 285.77972^\circ / 12^\circ \\ &= 23.81497 \end{aligned}$$

This means that 23 *tithis* are completed and the 24[th] *tithi* is running. Since 24 is greater than 15, the day falls in the *kṛṣṇa pakṣa* (dark fortnight). Subtracting 15 from 24, the *tithi* is the *navamī* of the *kṛṣṇa pakṣa*.

6.8 *NAKṢATRA*

The *nakṣatra* at a given time, on a given day, is the "asterism", one of the 27 divisions of the zodiac, from *Aśvinī* to *Revatī*, (see section 4.2), occupied by the *nirayana* (sidereal) moon.

The extent of a *nakṣatra* is $13°20' = 360°/27$, and hence the running *nakṣatra* is obtained by dividing the *nirayana* longitude of the Moon M by $13°20'$ (i.e., by $13.3333°$). Thus,

$$\begin{aligned} \textit{Nakṣatra} &= \text{(Longitude of the Moon in degrees)} / 13°.3333 \\ &= M/13.33333° \end{aligned}$$

Example: On March 21, 1990, at 5h 30m (IST), the *nirayana* longitude of the Moon is 8s $22°10'$.

This means that the longitude of the Moon, in degrees, is

$$M = 262.16666°$$

so that

$$\begin{aligned} \textit{Nakṣatra} &= M/13.33333° \\ &= 262.16666°/13.33333° = 19.6625. \end{aligned}$$

Therefore, 19 *nakṣatras* are completed and the 20[th] *nakṣatra*, viz. *Pūrvāṣāḍhā* (see Table 4.2), is ongoing at the given time.

6.9 *YOGA*

The sum of the *nirayana* longitudes of the Sun and the Moon is divided into 27 equal divisions called *yogas*.

The sum of the *nirayana* longitudes of the Sun and the Moon is converted into minutes and then divided by 800. The quotient represents the number of *yogas* completed and, hence, the current running *yoga* is obtained by adding 1 to the completed number of *yogas*.

There are 27 *yogas* as listed below:

1. *Viṣkambha*
2. *Prīti*
3. *Āyuṣmān*
4. *Saubhāgya*
5. *Śobhana*
6. *Atigaṇḍa*
7. *Sukarmā*
8. *Dhṛti*
9. *Sūla*
10. *Gaṇḍa*
11. *Vṛddhi*
12. *Dhruva*
13. *Vyāghāta*
14. *Harṣaṇa* (or *Vaidṛiti*)
15. *Vajra*
16. *Siddhi*
17. *Vyatīpāta*
18. *Varīyān*
19. *Parigha*
20. *Śiva*
21. *Siddha*
22. *Sādhya*
23. *Śubha*
24. *Śukla*
25. *Brahma*
26. *Indra*
27. *Vaidhṛta*

Note: If the sum of the longitudes of the Sun and the Moon (in degrees) *exceeds* 360°, then subtract 360° from the sum, convert into minutes and then divide that figure by 800.

Example: On March 21, 1990, at 5h 30m a.m. (IST), the longitude of the Sun is 336°23′ (neglecting the seconds) and the longitude of the Moon is 262°10′.

The sum of the longitudes of the Sun and the Moon is equal to $336°23' + 262°10' = 598°33'$.

Since the sum exceeds 360°, subtracting this value, the sum is:

$$\begin{aligned} S + M &= 598°\,33' - 360° \\ &= 238°\,33' \end{aligned}$$

Converting it into *minutes*, we have,

$$\begin{aligned} S + M &= (238° \times 60) + 33' \\ &= 14{,}313 \textit{ minutes} \end{aligned}$$

Therefore, the *yoga* is given by:

$$\begin{aligned} Y &= (S + M \text{ in } \textit{minutes}) \,/\, 800 \\ &= 14{,}313 \,/\, 800 \\ &= 17.89125 \end{aligned}$$

This means that 17 *yogas* are completed and the current *yoga* is the 18th one, viz., *Varīyān* (see the list). In fact, according to the *Rāṣṭrīya Pañcāṅga*, on that day the *Varīyān yoga* runs till 7h 59m a.m. (IST).

Note: If the sum of the *nirayana* longitudes of the Sun and the Moon is 180°, the phenomenon is called *Lata vyatīpāta* and if the sum is 360°, it is called *Vaidhṛta vyatīpāta*; and when the sum of the longitudes extends to the end of *Anurādhā nakṣatra*, i.e., if the sum is 226°40′, this phenomenon is called *Sārpamastaka vyatīpāta.*

6.10 *KARAṆA*

There are 11 *karaṇas* of which 4 are immovable and 7 are movable. The names of the *karaṇas* are as follows:

(i) *Cara* (movable) *karaṇas*: *Bava, Bālava, Kaulava, Taitila, Gara, Vanij, Viṣṭi.*

(ii) *Sthira* (immovable) *karaṇas*: *Śakuni, Catuṣpāda, Nāga and Kimstughna.*

In each *tithi*, the first half is one *karaṇa* and the second half is the next *karaṇa*, so that each *karaṇa* is situated at 6° of the angular distance between the Sun and the Moon. In the particular four half-*tithis*, viz., the second half of *kṛṣṇa pakṣa* (i.e. *Bahula*), *Caturdaṣī*, the two halves of *Amāvāsyā* and the first half of the *Pratipat* are the *sthira karaṇas*, viz., *Śakuni, Catuṣpāda, Nāga* and *Kimstughna* respectively. Then, from the second half of the *pratipat* of *śukla pakṣa*, we have the *cara karaṇas*, viz., *Bava*, etc., repeating the cycle of 7 *karaṇas*, eight times.

Note:

(i) If the longitude of the Moon M, is less than the longitude of the Sun S, then add 360° to $(M-S)$ before dividing it by 6°.

(ii) When $(M-S)$ is, divided by 6°, if the quotient (an integer) K is 57, 58, 59, 60 (or 0), then correspondingly, the *karaṇa* is *Śakuni, Catuṣpāda, Nāga* or *Kimstughna.*

(iii) If $(M-S)/6$ is less than 7, then let K be the quotient (i.e., the integral part). If $(M-S)/6$ is greater than 7, then subtract the nearest multiple of 7 from that number, and in the resulting number, let K be the integral part. Then K represents the *karaṇa* counting from *Bava* in the list of *cara karaṇas.*

Example 1: On March 21, 1990, at 5h 30m a.m. (IST) $M = 262°10'$ and $S = 336°23'13''$.

Since M is less than S, we add 360° to $(M - S)$ to get its new value,

$$(M - S) = 360° + (262°10' - 336°23') = 285.77972°$$

Now,

$$(M - S) / 6° = 285.77972 / 6 = 47.629953$$

Since this number is greater than 7, by removing the nearest multiple of 7 (namely, 42), we get 5.629953, so that $K = 5$. By counting to 5 starting with *Bava*, in the list of *cara karaṇas*, we obtain the running *karaṇa* as *Gara*.

Example 2: On August 2, 1989, at 5h 30m a.m. (IST), $M = 3s\ 19°52'$ and $S = 3s\ 15°58'$. Now,

$$(M - S) / 6° = (109.8666° - 105.9744°) / 6 = 0.6487033$$

so that $K = 0$. From item (ii) of the note above, the *karaṇa* is *Kimstughna*.

6.11 VĀRA

The remainder of the *pañcāṅga* is *vāra* or *vāsara*, i.e., the week day. A seven-day week is followed. The week days are *Ravi vāra, Soma vāra, Maṅgala vāra, Budha vāra, Guru vāra, Śukra vāra* and *Śani vāra* which correspond to Sunday, Monday, Tuesday, Wednesday, Thursday, Friday and Saturday. The seven days of the week are named after the seven luminaries: the Sun, Moon, Mars, Mercury, Jupiter, Venus and Saturn. This order has been obtained as follows. Arrange the 7 bodies in the decreasing order of their periods as Saturn, Jupiter, Mars, Sun, Venus, Mercury, Moon. Divide the day into 24 hours and assume that these bodies are the rulers of successive hours in cyclic order. Then the day is named after the ruler of the first hour. If we start from Saturn as the ruler of the 1st hour, it will again be the ruler of the 22nd hour. Then the 23rd hour will belong to Jupiter, 24th to Mars and 25th, i.e., the 1st hour of the next day to the Sun, and so on.

7

Mean Positions of the Sun, Moon and Planets

7.1 *AHARGAṆA*

For the purpose of finding the mean positions of the planets on any particular day, the total number of civil days elapsed since the beginning of the chosen epoch must first be determined. Then, this duration of time multiplied by the mean daily motion of a planet gives the distance covered by the planet during that period. From this motion the completed number of revolutions (multiples of 360°) is removed. The remainder when added to the mean position of the planet at the epoch gives the mean position for the required day.

The number of days elapsed since the chosen fixed epoch is called *ahargaṇa* which literally means a "heap of days". The calculation of the *ahargaṇa* depends on the calendar system followed. Since in the traditional Hindu calendar both the luni-solar calendar and the solar calendar, to which the former is pegged on to, are followed, the intercalary months (*adhikamāsa*) play an important role in calculating the *ahargaṇa*.

The process of determining the *ahargaṇa* essentially consists of the following steps:

(i) Convert the solar years elapsed (since the epoch) into lunar months;
(ii) Add the number of *adhikamāsas* during that period to give the actual number of lunar months that have elapsed upto the beginning of the current year;
(iii) Add the number of lunar months elapsed in the given year.
(iv) Convert these actually elapsed number of lunar months into *tithis* (by multiplying it by 30);
(v) Add the elapsed number of *tithis* in the current lunar month; and finally
(vi) Convert the elapsed number of *tithis* into civil days.

Note: While finding the *adhikamāsas*, if an *adhikamāsa* is due after the given lunar month of the current lunar year, then 1 is to be subtracted from the calculated number of *adhikamāsas*. This is because an *adhikamāsa* which is yet to come in the course of the current year will have already been added.

7.2 Working Method to Find the *Ahargaṇa*

Before evolving a working algorithm to find the *ahargaṇa*, we shall list some useful data for the purpose, according to the **Sūryasiddhānta**. In a *Mahāyuga* of 432×10^4 years, we have

(i) Number of sidereal revolutions of the Moon : 577,53,336

(ii) Number of revolutions of the Sun : 43,20,000

(iii) Number of lunar months in a *Mahāyuga* of 432×10^4 years [*(i)* – *(ii)*] : 534,33,336

(iv) Number of *adhikamāsas* in a *Mahāyuga*

$$= \text{Number of lunar months} - (12 \times \text{Number of solar years})$$
$$= 534{,}33{,}336 - (12 \times 43{,}20{,}000)$$
$$= 534{,}33{,}336 - 518{,}40{,}000$$
$$= 15{,}93{,}336$$

Suppose we want to find the *ahargaṇa* for the day on which "x" luni-solar years, "y" lunar months and "z" *tithis* have elapsed.

Then, we have the number of *adhikamāsas* in x completed luni-solar years given by

$$x_1 = \text{INT}\,[\,(x)\,(15{,}93{,}336 \,/\, 43{,}20{,}000)\,]$$

where INT (i.e., integer value) means that only the quotient of the expression in brackets is considered. Now, since in the current luni-solar year, y lunar months and z *tithis* have elapsed, we have the following:

Number of lunar months elapsed since the epoch

$$= 12x + x_1 + y + z \,/\, 30$$

where the number of elapsed *tithis* z is converted into a fraction of a lunar month.

The average duration of a lunar month is 29.530589 days. Therefore, the number of civil days N^1 elapsed since the epoch began are,

$$N^1 = \text{INT}\,[\,(12x + x_1 + y + z / 30) \times 29.530589\,] + 1$$

Here also, only the integer part of the expression in square brackets is considered.

Since, in our calculations, we have considered only the mean duration of a lunar month, the result may have a maximum error of 1 day. Therefore, to get the actual *ahargaṇa* N, an addition to or a subtraction from N^1 of 1 may be necessary.

This is decided by the verification of the week day. The tentative *ahargaṇa* N^1 is divided by 7 and the remainder is expected to give the week day, when counted from the week day of the chosen epoch. For example, the epoch of *Kaliyuga* is known to have been a Friday. Therefore, when N^1 is divided by 7, if the remainder is 0, then that day must be a Friday, if 1 then Saturday, etc. However, if the calculated week day is different from the actual week day, then 1 is either added to or subtracted from N^1, so as to get the same calculated and actual week day. Then, accordingly, the actual *ahargaṇa* $N = N^1 \pm 1$.

It is important to note that the method described above is a simplified version of the actual procedure described variously by various Siddhāntic texts.

Note: While determining the number of *adhikamāsas* x_1 by the aforesaid method, if an *adhikamāsa* is due after the given lunar month in the said lunar year, then subtract 1 from x_1 to get the correct number of *adhikamāsas.*

Example: Find the *Kali ahargaṇa* corresponding to *Caitra kṛṣṇa trayodasi* of the *śaka* year 1913 (elapsed), i.e., for April 12, 1991.

Number of *Kali* years = 3179+1913=5092

since the beginning of *Śaka*, i.e., 78 AD corresponds to 3179 years (elapsed) of *Kali*. Therefore,

Number of *adhikamāsas* in 5092 years

$$= (15{,}93{,}336 \,/\, 43{,}20{,}000) \times 5092\,'' = 1878.0710$$

Taking the integral part of the above value, $x_1 = 1878$.

Now, an *adhikamāsa* is due just after the *Caitra māsa* under consideration. Although the *adhikamāsa* is yet to occur, it has already been included in the above value of x_1. Therefore,

Corrected value of $x_1 = 1877$

Since the month under consideration is *Caitra*, the number of elapsed lunar months in the lunar year is given by $y=0$. The current *tithi* is the *trayodasi* of *kṛṣṇa pakṣa*, so that the elapsed number of *tithis* is $15+12=27$, i.e., $z=27$. Therefore,

Number of lunar months completed

$$= (5092 \times 12) + 1877 + 0 + 27 / 30$$
$$= 62{,}981.9$$

(Tentative) number of civil days,

$$N^1 = \text{INT}\,[62{,}981.9 \times 29.530589] + 1$$
$$= \text{INT}\,[18{,}59{,}892.603] + 1$$
$$= 18{,}59{,}893.$$

Now dividing N^1 by 7, the remainder is 0 counting 0 as Friday, 1 as Saturday, etc., the remainder corresponds to Friday. Also, from the calendar, April 12, 1991, was a Friday. Therefore, we have the *ahargaṇa*,

$N = 18{,}59{,}893$

since the *Kali* epoch.

7.3 Mean Positions-Computations

It was pointed out, while discussing the *yuga* system of Indian astronomy, that the mean motion of each planet is given in the number of revolutions made by the planet in the course of a *Mahāyuga* of 432×10^4 years, or a *Kalpa* of 432×10^7 years. The advantage of choosing such a long period of time is that the motions of planets could be expressed in an *integral* number of revolutions completed, avoiding inconvenient fractions. The *yuga* model is used to compute the mean positions of planets. These positions are then corrected to get the true positions.

The following is the usual procedure for finding the mean position of any planet: Let λ_0 be the mean position at an epoch. If d is the daily mean motion of the planet in degrees and A is the *ahargaṇa* from the epoch, then the mean motion from the epoch till the given day is $\lambda_1 = (A \times d)$. Then, the mean position on the given day is

$$\lambda = \lambda_0 + \lambda_1 = \lambda_0 + (A \times d)$$

In the texts where the beginning of *Kaliyuga* is used as the epoch, the *ahargaṇa* is calculated from the midnight between the 17th and 18th of February 3102 BC. Therefore, in this method, the *ahargaṇa* A is obtained till the midnight preceding the given day. Further, the midnight is as at the Ujjayinī (23°11′ N latitude and 75°46′ E longitude) meridian passing through Laṅkā on the equator.

Therefore, when we calculate the positions of the planets, corrections will have to be applied to account for:

(i) the time interval between the midnight at Ujjayinī and the midnight at the given place;

(ii) the difference in duration from the local midnight to the given time.

In addition to these, some more important corrections will also have to be applied. These will be discussed later.

Some astronomical texts consider the mean *sunrise* at Laṅkā on February 18,3102 BC as the starting time of the *Kali* era.

The mean motion of the Sun, the Moon and other planets are given in terms of the number of revolutions (each of 360° extent) completed in the course of a *Kalpa* of 432×10^7 years. Table 7.1 gives the necessary details for determining the mean positions of the Sun and the Moon according to different texts. For the other planets, the details will be considered later.

Looking at Table 7.1, we notice that while the revolutions of the Sun in a *Kalpa* are the same according to the different texts, those of the Moon and its apogee and node (*Rāhu*) are different. Further, the number of civil days in a *Mahāyuga* (or multiplied by 1000 for a *Kalpa*) are slightly different according to Āryabhaṭa I and the **Sūrya-siddhānta**, for example. These differences have resulted from the corrections made periodically, and give rise to slightly different mean

Table 7.1 Revolutions of the Sun and the Moon in a *Kalpa*
(1 *Kalpa* = 432×10^7 years = 1000 *Mahāyugas*)

Bodies and points	*Ravi*	*Candra*	*Candra*'s Apogee	*Candra*'s (Asc.) Node	Civil days in a *Mahāyuga*
Āryabhaṭa I	432,00,00,000	5775,33,36,000	48,82,19,000	23,21,16,000	1,57,79,17,500
Brahmagupta (**Khaṇḍakhādyaka**)	432,00,00,000	5775,33,36,999	48,82,19,000	23,22,26,000	1,57,79,17,800
Sūryasiddhānta	432,00,00,000	5775,33,36,000	48,82,03,000	23,22,38,000	1,57,79,17,828
Āryabhaṭa II (**Mahāsiddhānta**)	432,00,00,000	5775,33,34,000	48,82,08,674	23,23,13,354	
Bhāskara II (**Siddhāntaśiromaṇi**)	432,00,00,000	5775,33,00,000	48,82,05,858	23,23,11,168	1,57,79,16,450

daily motions. The mean daily motions are given in Table 7.2 according to the **Siddhāntaśiromaṇi** of Bhāskara II, as compared to the modern values, and those of the **Khaṇḍakhādyaka** of Brahmagupta and the **Sūryasiddhānta**.

Table 7.2 Daily mean motion of the Sun, the Moon, etc.

Bodies and points	**Sūrya Siddhānta**	*Siddhānta-śiromaṇi*	**Modern Astronomy**	**Khaṇḍa-Khādyaka**
Ravi	0°59 08 10 09.7	0°59 08 10 21	0°59 08.2	0°59 08
Candra	13°10 34 52 02	13°10 34 53 00	13°10 34.9	13°10 31
Kuja	0°31 26 28 10	0°31 26 28 07	0°31 26.5	0°31 26
Budha's śīghrocca	4°05 32 20 42	4°05 32 18 28	4°05 32.4	4°05 32
Śukra's śīghrocca	1°36 07 43 37	1°36 07 44 35	1°36 07.7	1°36 07
Guru	0°04 59 08 48	0°04 59 09 09	0°04 59.1	0°04 59
Śani	0°02 00 22 53	0°02 00 22 51	0°02 00.5	0°02 00
Candra's mandocca	0°06 40 58 42	0°06 40 53 56	0°06 40.92	0°06 40
Candra's pāta (Rāhu)	–0°03 10 44 43	–0°03 10 48 20	–0°03 10.77	–0°03 10

It is clear that the daily motion of a celestial body is given, in revolutions per day, by

$$d = \frac{\text{Number of revolutions in a } \textit{Mahāyuga}}{\text{Number of civil days in a } \textit{Mahāyuga}}$$

For example, according to the **Siddhāntaśiromaṇi**, the number of revolutions completed by the Moon is 577,53,300 in a *Mahāyuga* that has 157,79,16,450 civil days. Therefore, the Moon's daily motion is,

$$\begin{aligned} d_{\text{Moon}} &= (577{,}53{,}300 \ / \ 157, 79{,}16{,}450) \times 360 \text{ (in degrees)} \\ &= 13^\circ\ 10'\ 34''.8796 \end{aligned}$$

Similarly, for the Sun, the mean daily motion is,

$$\begin{aligned} d_{\text{Sun}} &= (43{,}20{,}000 \ / \ 157{,}79{,}16{,}450) \times 360 \text{ (in degrees)} \\ &= (43{,}20{,}000 \times 360 \ / \ 157{,}79{,}16{,}450) \text{ degrees} \\ &= 0^\circ\ 59'\ 08''.1726 \end{aligned}$$

Now, as explained earlier, the mean motion for any given day is obtained by multiplying the mean daily motion of the celestial body by the *ahargaṇa* of the day, that is,

$$\lambda_1 = d \times A$$

The mean motion λ_1, thus obtained, must be added to the initial mean position at the epoch, viz., λ_0. According to Bhāskara II, the initial mean positions (*dhruvakas*), λ_0 for the different celestial bodies are shown in Table 7.3. The values according to the **Sūryasiddhānta** are also given.

Table 7.3 Mean positions of planets at the *Kali* epoch

Planets and points	Siddhāntaśiromaṇi	Sūryasiddhānta
Ravi	0°0 0	0°0 0
Candra	0°0 0	0°0 0
Kuja	359°03 50	0°0 0
Budha śīghrocca	357°24 29	0°0 0
Guru	359°27 36	0°0 0
Śukra śīghrocca	358°42 14	0°0 0
Śani	358°46 34	0°0 0
Ravi's mandocca	77°45 36	77°7 48
Candra's mandocca	125°29 46	90°0 0
Candra's pāta (*Rāhu*)	153°12 58	180°0 0

For the computation of the mean positions of the planets, we chose the beginning of the *Kaliyuga*, viz., the midnight of 17/18 February 3102 BC, as the epoch. Further, it is assumed that the Sun, the Moon and the five planets (*tārāgrahas*) were at the beginning of the zodiac, i.e., 0° *Meṣa*. The mean longitude of a planet at the mean midnight at Ujjayanī on a day with *Kali ahargaṇa A* is given by:

$$\begin{aligned} \text{Mean longitude} &= A \times \text{Mean daily motion} \\ &= A \times \text{No. of revolutions / No. of civil days} \end{aligned}$$

where the number of revolutions of the concerned body and the number of civil days correspond to a *Mahāyuga* of 43,20,000 years.

(i) MEAN LONGITUDE OF THE SUN

In a *Mahāyuga*, the number of revolutions of the Sun are 43,20,000 and the number of civil days are 157,79,17,828.

Now, on March 22, 1991, the elapsed *Kali ahargaṇa* = 18,59,872. Therefore,

Mean longitude of the Sun

$$= 18{,}59{,}872 \times 43{,}20{,}000 \,/\, 157{,}79{,}17{,}828$$
$$= 8.03464 \times 10^{12} \,/\, 157{,}79{,}17{,}828$$
$$= 5091 \text{ rev. } 334°46'49''$$

Now, ignoring the completed revolutions, viz., 5091, the mean longitude of the Sun is 334°46′49″

(ii) MEAN LONGITUDE OF THE MOON

In a *Mahāyuga*, the number of revolutions of the Moon are 577,53,336 and the number of civil days are 157,79,17,828. At the midnight of March 22, 1991, the elapsed *ahargaṇa* is 18,59,872.
So we have:

Mean longitude of the Moon

$$= 18{,}59{,}872 \times 577{,}53{,}336 \,/\, 157{,}79{,}17{,}828$$
$$= 1.07413 \times 1014 \,/\, 157{,}79{,}17{,}828$$
$$= 68{,}073 \text{ rev. } 48°25'9''.$$

Ignoring the number of completed revolutions, the mean longitude of the Moon is 48°25′9″.

Note: In the above computations, since the mean longitudes are given in terms of revolutions, the decimal part is multiplied by 360° to get it in degrees, the decimal part of the degree by 60 to get minutes and lastly the decimal part of the minute by 60 to get seconds.

(iii) MEAN LONGITUDE OF THE MOON'S APOGEE (*MANDOCCA*)

In a *Mahāyuga*, the number of revolutions of the Moon's *mandocca* is 48,82,03; and the number of civil days is 157,79,17,828. At the epoch, the *mandocca* was 90°. Therefore, for the *ahargaṇa* of 18,59,872,

the mean motion of the Moon's *mandocca*

$$= 48{,}82{,}03 \times 18{,}59{,}872 \,/\, 157{,}79{,}17{,}828$$
$$= 9.07995 \times 10^{11} \,/\, 157{,}79{,}17{,}828$$
$$= 575.43878 \text{ rev.}$$
$$= 575 \text{ rev. } 157°57'22''.$$

Removing the number of completed revolutions, the mean motion of the Moon's *mandocca* since the epoch is $157°57'22''$. Now, adding this mean motion to the longitude of the *mandocca* at the epoch, we have the mean longitude of the Moon's *mandocca*

$$= 90° + 157°57'22'' = 247°57'22''$$

(iv) MEAN LONGITUDE OF THE MOON'S NODE (*RĀHU*)

The number of revolutions of the ascending node of the Moon, called *Rāhu*, in a *Mahāyuga* is 2,32,238. *Rāhu* was assumed to be at 180° from the beginning of the zodiac at the epoch, i.e., the beginning of the *Kaliyuga*. Therefore,

Mean longitude of *Rāhu* = 180° – mean motion for the *ahargaṇa*.

[*Note:* The mean motion is subtractive since *Rāhu* moves backwards along the ecliptic.]

In the example, we have

Mean motion of *Rāhu*

$$\begin{aligned} &= (\text{No. of revolutions / No. of civil days}) \times \textit{ahargaṇa} \\ &= (2{,}32{,}238 \,/\, 157{,}79{,}17{,}828) \times 18{,}59{,}872 \\ &= 4.31932 \times 10^{11} / \, 157{,}79{,}17{,}828 \\ &= 273.7603 \text{ rev.} \\ &= 273 \text{ rev. } 264°\ 58'\ 8'' \end{aligned}$$

After removing the completed number of revolutions, the motion of *Rāhu* is $264°58'8''$. Since the motion is backwards, this value has to be subtracted from the initial position at the epoch, i.e., 180°. Therefore,

$$\begin{aligned} \text{Mean longitude of } \textit{Rāhu} &= 180° - (264°58'8'') \\ &= 275°01'52''. \end{aligned}$$

by adding 360° to avoid the negative value.

Note: The opposite node, called the descending node of the Moon (*Ketu*), is exactly 180° away from *Rāhu*. Therefore,

Mean longitude of *Ketu* = $95°01'52''$.

7.4 MEAN POSITIONS OF THE STAR-PLANETS (*KUJA, BUDHA, GURU, ŚUKRA* AND *ŚANI*)

The mean positions of the three *tārāgrahas* ("star-planets") namely, *Kuja, Guru* and *Śani* are determined by the same procedure as in the case of the Sun and the Moon, while the mean positions of *Budha* and *Śukra* are taken as equal to that of the Sun. But in their case we compute the mean positions of their *śīghrocca*. This aspect will be discussed in the next chapter.

Table 7.4 Revolutions of planet's in a *Mahāyuga* (Sūryasiddhānta)
(Number of civil days in a *Mahāyuga* : 157,79,17,828)

Planet	No. of revolutions	Mean daily motion (*d*)
Kuja	22,96,832	0.5240193°
Budha śīghrocca	1,79,37,060	4.0923181°
Guru	3,64,220	0.0830963°
Śukra śīghrocca	70,22,376	1.6021464°
Śani	1,46,568	0.0334393°

Note: In actual numerical computations, to avoid 'rounding off' errors, double precision decimal representation of numbers can be used on computers. In the following computations, there is a small amount of rounding off error.

Example: Find the mean longitudes of the five planets at the midnight between March 21 and 22, 1991.
Now, the *ahargaṇa* elapsed : 18,59,872.

(1) Mean longitude of *Kuja*

$$= \textit{ahargaṇa} \times \text{mean daily motion}$$
$$= 1859872 \times 0.5240193^\circ$$
$$= 974608.82^\circ = 88^\circ 49' 12''$$

(after removing the nearest integral multiple of 360°)

(2) Mean longitude of *Budha's śīghrocca*

$$= 1859872 \times 4.0923181^\circ$$
$$= 7611187.8^\circ = 67^\circ 48'$$

(after removing the nearest integral multiple of 360°)

(3) Mean longitude of *Guru*

$= 1859872 \times 0.0830963°$

$= 154548.48° = 108°28'48''$

(after removing the nearest integral multiple of 360°)

(4) Mean longitude of *Śukra's śīghrocca*

$= 1859872 \times 1.6021464°$

$= 2979787.2° = 67°12'$

(after removing the nearest integral multiple of 360°)

(5) Mean longitude of *Śani*

$= 1859872 \times 0.0334393°$

$= 62192.818° = 272°49'5''$

(after removing the nearest integral multiple of 360°)

7.5 *DEŚĀNTARA* CORRECTION

Deśāntara is the longitude of a place measured to the east or west from the standard (or prime) meridian. In Indian astronomy, the prime meridian is the great circle of the Earth passing through the north and south poles, Ujjayinī and Laṅkā, where Laṅkā was assumed to be on the Earth's equator.

The Sun completes a rotation (of 360°) in one day due to the diurnal motion. Therefore, the angular distance covered by the Sun in 1 hour is 360/24, i.e., 15°. This means if the Sun rises at 6 a.m. (IST) at a place A, then the sunrise at another place B, 15° to the *west* of the first place will be 1 hour later, i.e., at 7 a.m. (IST), provided that the two places are on the same latitude (i.e., on the same small circle parallel to the equator). Similarly, at a place C to the *east* of A, the sunrise will have taken place earlier. If C is 15° to the east of A (but on the same small circle parallel to the equator), then the sunrise at C will be 1 hour before that at A, i.e., 5 a.m. (IST)

In Indian astronomy, the mean positions of planets are calculated either at midnight or at sunrise at the standard meridian. Then, to get the position at the corresponding time (i.e., either midnight or sunrise) at another place, on a different meridian, we have to apply a correction due to the difference in the longitudes of the given place and the standard meridian. This correction is called the *deśāntara* correction.

Since the standard meridian passes through Ujjayinī, in terms of the modern terrestrial longitudes (with reference to the Greenwich meridian), we have

Deśāntara = [Long. of the place – Long. of Ujjayinī] / 15

Since the longitude of Ujjayinī is $75^\circ E45'$ (east of Greenwich), we have

Deśāntara = [Long. of the place – $75^\circ 45'$] / 15 in *hours* ... (1)

If the correction is required in *ghaṭikās*, since the Earth rotates at the rate of 360° per 60 *ghaṭikās* (or 6° per *gh.*), we have

Deśāntara = [Long. of the place – $75^\circ 45'$] / 6 *gh.* ... (2)

The correction to be applied to the mean Sun or Moon or any planet (obtained for Ujjayinī midnight) due to *deśāntara* is given by:

Deśāntara correction = (*Deśāntara* in hours) × d / 24
= (*Deśāntara* in *ghaṭikās*) × d / 60 ... (3)

where d is the mean *daily* motion of the Sun or the Moon or the planet, as the case may be.

The above correction is to be added to or subtracted from the earlier obtained mean position (for Ujjayinī at midnight), according to whether the place is to the west or east of Ujjayinī.

If the place is to the *west* of Ujjayinī, midnight occurs there later than at Ujjayinī, during which time-interval the celestial body will have moved further; hence, the *deśāntara* correction in that case is *additive.*

Similarly, if the place is to the *east* of Ujjayinī, midnight there would have occured earlier and hence the celestial body will have moved less; therefore, the correction is *subtractive.* Thus, from (1), (2) and (3) we have

Deśāntara correction = $-(\lambda - \lambda_0) \times d / 360$

where

λ = Longitude of the place
λ = Longitude of Ujjayinī and
d = Mean daily motion of a celestial body.

The negative sign indicates that the correction is subtractive when $\lambda > \lambda_0$, i.e., the place is to the east of Ujjayinī. For a place lying to the west of Ujjayinī (i.e., $\lambda < \lambda_0$), the correction automatically becomes additive. Note that for a place with a western longitude (with reference to Greenwich), λ must be taken as negative.

Note: In the Indian astronomical texts, the *deśāntara* is obtained from the linear distance (in *yojanas*) of the place from Ujjayinī.

Example: We shall now apply the *deśāntara* corrections to the Sun, the Moon, the Moon's apogee and the node for Bangalore at the local midnight between 21st and 22nd March, 1991. Taking the longitudes of Bangalore and Ujjayinī as 77°35′ and 75°47′, we have

$$(\lambda - \lambda_0) / 360 = 1.8 / 360$$

(i) *DEŚĀNTARA* CORRECTION FOR THE SUN

Since the daily mean motion of the Sun is 59.136078′, according to the **Sūryasiddhānta**, we have:

$$\begin{aligned} \textit{Deśāntara} \text{ correction for the Sun} &= (1.8 / 360) \times 59.136078' \\ &= 17.7'' \end{aligned}$$

Therefore, the mean longitude of the Sun at the local mean midnight of Bangalore is $334°46'49'' - 17.7' = 334°46'31''$

(ii) *DEŚĀNTARA* CORRECTION FOR THE MOON

Daily mean motion of the Moon: 13.176352°. Therefore,

$$\begin{aligned} \textit{Deśāntara} \text{ correction} &= (1.8 / 360) \times 13.176352° \\ &= 3'57'' \end{aligned}$$

Mean longitude of the Moon at the local midnight of Bangalore is,

$$48°25'9'' - 3'57'' = 48°21'12''$$

(iii) *DEŚĀNTARA* CORRECTION FOR THE MOON'S APOGEE

Daily mean motion of the Moon's apogee is 6.6829747′. Therefore, for the Moon's apogee,

$$\begin{aligned} \textit{Deśāntara} \text{ correction} &= (1.8 / 360) \times 6.6829747 \\ &= 2'' \end{aligned}$$

The mean longitude of the Moon's apogee at the mean midnight at Bangalore is

$$247°57'22'' - 2'' = 247°57'20''.$$

(iv) *DEŚĀNTARA* CORRECTION FOR THE MOON'S NODE

Daily mean motion of the Moon's node is $-0.052985°$ or $-190.746''$ so that

$$\begin{aligned} \textit{Deśāntara}\text{ correction} &= -(1.8 / 360) \times 190.746'' \\ &= -0.95373'' \\ &= -1''\text{ approximately} \end{aligned}$$

Thus, the mean longitude of *Rāhu* for the local mean midnight of Bangalore is

$$275°01'52'' - (-1'') = 275°01'53''.$$

7.6 *DEŚĀNTARA* CORRECTION FOR THE STAR-PLANETS

Example: Apply the *deśāntara* correction to the five planets for the midnight preceding March 22, 1991 at Bangalore. Now, as explained in section 7.5, we have

Longitude of Ujjayinī, $\lambda_0 = 75°\,47'$ E
Longitude of Bangalore, $\lambda = 77°\,35'$ E

so that

$$(\lambda - \lambda_0) / 360 = 1.8 / 360$$

If this factor is multiplied by the mean daily motion d of a planet, we obtain the *deśāntara* correction for that planet and it is subtractive since $\lambda > \lambda_0$.

We shall calculate this correction and apply it to each of the five planets as shown below, using d as given in Table 7.4.

(1) *DEŚĀNTARA* CORRECTION FOR *KUJA*

$$\begin{aligned} &= -(1.8 / 360) \times 0.5240193° \\ &= -0°0'09'' \end{aligned}$$

Therefore, the mean longitude of *Kuja* at the midnight of Bangalore: $88°49'12'' - 09'' = 88°49'03''$.

(2) *DEŚĀNTARA* CORRECTION FOR *BUDHA'S ŚĪGHROCCA*

$$= -(1.8 / 360) \times 4.0923181° = -0°01'14''$$

Therefore, the mean longitude of *Budha's śīghrocca* at Bangalore's midnight: $67°48' - 1'14'' = 67°46'46''$

(3) *DEŚĀNTARA* CORRECTION FOR *GURU*

$$= -(1.8 / 360) \times 0.0830963° = -0°0'1.5''$$

Therefore, the mean longitude of *Guru* at the midnight of Bangalore: $108°28'48'' - 1.5'' = 108°28'47''$.

(4) *DEŚĀNTARA* CORRECTION FOR *ŚUKRA'S ŚĪGHROCCA*

$$= -(1.8 / 360) \times 1.6021464° = -0°0'29''$$

Therefore, the mean longitude of *Śukra's śīghrocca* at Bangalore's midnight : $67°12' - 29'' = 67°11'31''$.

(5) *DEŚĀNTARA* CORRECTION FOR *ŚANI*

$$= -(1.8 / 360) \times 0.0334393° = -0°0'0.6'' \approx -1''$$

Therefore, the mean longitude of *Śani* at midnight in Bangalore: $272°49'5'' - 1 = 272°49'4''$.

8

True Positions of the Sun and the Moon

8.1 Introduction

To obtain the mean positions of the Sun and the Moon, it was assumed that these bodies move in circular orbits around the Earth with uniform angular velocities. However, by observation it was found that the motions are not uniform.

The procedure for calculating the major corrections to the mean positions, to obtain the true positions, is related to the epicyclic theory* which is explained in the following section.

8.2 Epicyclic Theory – *Mandaphala*

The theory is that while the *mean* Sun or Moon moves along a big circular orbit of radius R (dotted in Fig.8.1), the actual (or *true*) Sun or Moon moves along another smaller circle of radius r, called *epicycle*, whose centre is on the bigger circle.

The bigger circle ABP, with the Earth as its centre, is called the *kakṣavṛtta*. Let A be position of the mean Sun when the true Sun is farthest from the Earth. The line AEP is called the *apse line* (or *nicoccarekhā*). The epicycle, with A as its centre and a prescribed radius r (smaller than AE), is called the *nicoccavṛtta*. Let the apse line PEA cut the epicycle at U and N. The two points U and N are respectively called the *mandocca* (apogee) and the *mandanīca* of the the Sun. Note that as the Sun moves along the epicycle, it is farthest from the Earth when it is at U and nearest when at N.

* It is to be noted that this theory is only an approximation to the true elliptical orbits of the places.

The epicyclic theory assumes that as the centre of the epicycle (i.e., mean Sun) moves along the circle *ABP* in the direction of the signs (from west to east) with the velocity of the mean Sun, the true Sun itself moves along the epicycle with the same velocity but in the opposite direction (from east to west). Further, the time taken by the true Sun to complete one revolution along the epicycle is the same as that taken by the mean Sun (i.e., centre of the epicycle) to complete a revolution along the orbit.

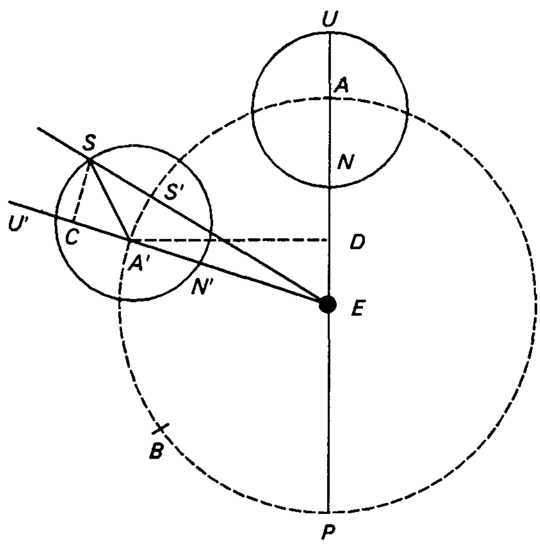

Figure 8.1 Epicyclic theory

Now, in Fig. 8.1, suppose the mean Sun moves from A to A'. Let $A'E$ be joined, cutting the epicycle at U' and N' which are the current positions of the apogee and the perigee. While the mean Sun is at A', suppose the true Sun is at S on the epicycle, so that $U'\hat{A}'S = U'\hat{E}A$. Join ES, cutting the orbit (i.e., circle *ABP*) at S'. Then A' is the *madhya* (mean Sun) and S' is *spaṣṭa* (or *sphuṭa*) *Ravi*. The difference between the two positions, viz., $A'\hat{E}S'$ (or arc $A'S'$) is called the *equation of centre* (or *mandaphala*).

Now, in order to obtain the true position of the Sun, it is necessary to get an expression for the equation of the centre which will have to be applied to the mean position.

In Fig. 8.1, SC and $A'D$ are drawn perpendicular to $U'N'E$ and UNE, respectively. The arc AA' (or $A\hat{E}A'$), the angle between the mean Sun and the apogee is called the mean *anomaly* m of the Sun (*mandakendra*).

We have, in the right-angled triangle $A'DE$,

$$\sin A\hat{E}A' = \sin D\hat{E}A' = A'D \,/\, A'E$$

so that

$$A'D = \mathrm{R} \sin AA' = \mathrm{R} \sin m.$$

$A'D$ is called the *mandakendrajyā*. From the similar right-angled triangles SCA' and $A'DE$, we have

$$SC \,/\, SA' = A'D \,/\, A'E$$

so that

$$SC = A'D \times SA' \,/\, A'E = (\mathrm{r} \,/\, \mathrm{R}) \sin m$$

Since SA' and $A'E$ are, respectively, the radii of the epicycle and the orbit, and these are proportional to the circumferences of the two circles; that is,

$$\begin{aligned} SA' \,/\, A'E &= \text{Circumference of epicycle} \,/\, \text{Circumference of orbit} \\ &= 2\pi r \,/\, 2\pi R \end{aligned}$$

so that

$$SC = (\text{Circumference of epicycle/Circumference of orbit}) \times A'D$$

Taking the circumference of the orbit as 360°, we have $R = 3438'$.

$$\therefore\ SC = (\text{Circumference of epicycle in degrees}) \times mandakendrajy\bar{a} \,/\, 360^\circ.$$

Now, taking SC as approximately the same as $\mathrm{A'S'}$, we have the *Equation of centre*

$$\begin{aligned} &= (\text{Circumference of epicycle})(mandakendrajy\bar{a}) \,/\, 360^\circ \\ &= (r/R)\,(R \sin m). \end{aligned}$$

where $R \sin (m)$ is the "Indian sine" of the anomaly of the Sun m. The maximum value of the equation of centre is r, the radius of the epicycle in degrees. By observation, this can be obtained as the maximum

deviation of the Sun's position from the calculated mean position. Note that when the Sun is at its *apogee* or *perigee*, the mean and true positions coincide since sin (m) is 0, when $m = 0°$ or $180°$.

The maximum equation of the centre for the Sun was observed by Bhāskara II, to be $2° \, 11' \, 30''$ which is the value of r. Therefore,

the circumference of the epicycle of the Sun
$$= (131.5' / 3438) \times 360° = 13.66°$$

This value was given by Bhāskara II.

Note: The same epicyclic theory is applied to the Moon also. In the case of the Moon, Bhāskara II has given the maximum equation of the centre as $302'$. Most texts have taken the epicycles as being of varying radii and not fixed.

From Table 8.1 we notice that while the **Khaṇḍakhādyaka** and the **Saurasiddhānta** (as given by Varāhamihira) take the epicycles as having a constant periphery (and hence radius), the **Āryabhaṭiyam** and the later **Sūryasiddhānta** take them as varying between two limits.

Table 8.1 Peripheries of epicycles of Apsis

Bodies	**Āryabhaṭiyam**	**Khaṇḍa-khādyaka**	**Saura-siddhānta (Varāhamihira)**	**Sūryasiddhānta**
Ravi	13°30′	14°	14°	13.66° to 14°
Candra	31°30′	31°	31°	31.66° to 32°
Kuja	63.0° to 81.0°	70°	70°	72° to 75°
Budha	22.5° to 31.5°	28°	28°	28° to 30°
Guru	31.5° to 36.5°	32°	32°	32° to 33°
Śukra	9.0° to 18.0°	14°	14°	11° to 12°
Śani	40.5° to 58.5°	60°	60°	48° to 49°

8.3 Equation of Centre (*Mandaphala*) for the Sun and the Moon

Now, how are these peripheries of the epicycles used to determine the equations of the centre or *mandhaphala*? We will follow the procedure given by **Sūryasiddhānta**.

For example, in the case of the Sun, the periphery varies from $13°$ $40'$ to $14°$. Therefore, the radius r varies from $(13°40' / 360°) \times 3438'$ to $(14° / 360°) \times 3438'$, i.e., from $130.517'$ to $133.7'$. But then we must know how to find the actual value of r at a given moment, between the given limits. For this, **Sūryasiddhānta** gives the following rule: "The degrees of the Sun's epicycle of the apsis (*manda paridhi*) are fourteen, ... at the end of the even quadrants; and at the end of the odd quadrants, they are twenty minutes less.."*

There are four quadrants; the *odd* quadrant endings are $90°$ and $270°$, and the even *quadrant* endings are $180°$ and $360°$ (or $0°$). Let m be the mean anomaly (*mandakendra*) of the Sun where

$$m = \textit{Mandocca} \text{ of the Sun} - \text{mean longitude of the Sun}$$

Since at $m = 90°$ and $m = 270°$, the periphery is minimum and at $m = 0°$ and $m = 180°$ it is maximum (i.e., $14°$), we can formulate:

$$\text{corrected periphery} = 14° - (20 \times |\sin m|)$$

assuming that the variation is sinusoidally periodic. Correspondingly, we have the corrected radius of the epicycle of the Sun's apsis:

$$r = (3438' / 360°)\,[14° - (^1/_3)°\,|\sin m|]$$

Similarly, in the case of the Moon, the periphery of the epicycle varies from $31.66°$ to $32°$ Hence, the corrected radius is given by

$$r = (3438' / 360°)\,[32 - (^1/_3)\,|\sin m|]$$

where

$$m = \text{Moon's } \textit{mandocca} - \text{Moon's mean longitude.}$$

Having found out the corrected radius of the epicycle, the

$$\textit{Mandaphala} = r \sin m$$

so that with the corrected r, we have the following:

$$\begin{aligned}\text{Sun's } \textit{mandaphala} &= (3438 / 360)\,[14° - (^1/_3)°\,|\sin \mathrm{m}|]\sin m \\ &= [133.7 \sin m - 3.183\,(\sin m).\,|\sin m|]'\end{aligned}$$

* There is no need for this device in modern astronomy.

$$\text{Moon's } mandaphala = (3438 / 360)\,[32^\circ - ({}^1/_3)^\circ\,|\sin m|\,]\,\sin m$$
$$= [305.6 \sin m - 3.183\,(\sin m)\,|\sin m|\,]'$$

The *mandaphala* is *additive* for $m < 180^\circ$, and *subtractive* for $m > 180^\circ$.

Table 8.2 Sines according to the **Sūryasiddhānta** (R=3438 and R = 3437.75)

S.No.	Arc(ϑ)	Arc (ϑ) (min.)	R sin (ϑ) (Hindu)	Difference	R'sin ϑ (True)
1.	3° 45′	225	225		224.84
2.	7° 30′	450	449	224	448.72
3.	11° 15′	675	671	222	670.67
4.	15° 00′	900	890	219	889.76
5.	18° 45′	1125	1105	215	1105.03
6.	22° 30′	1350	1315	210	1315.57
7.	26° 15′	1575	1520	205	1520.48
8.	30° 00′	1800	1719	199	1718.88
9.	33° 45′	2025	1910	191	1909.91
10.	37° 30′	2250	2093	183	2092.77
11.	41° 15′	2475	2267	174	2266.67
12.	45° 00′	2700	2431	164	2430.86
13.	48° 45′	2925	2585	154	2584.64
14.	52° 30′	3150	2728	143	2727.35
15.	56° 15′	3375	2859	131	2858.38
16.	60° 00′	3600	2978	119	2977.18
17.	63° 45′	3825	3084	106	3083.22
18.	67° 30′	4050	3177	93	3176.07
19.	71° 15′	4275	3256	79	3255.31
20.	75° 00′	4500	3321	65	3320.61
21.	78° 45′	4725	3372	51	3371.70
22.	82° 30′	4950	3409	37	3408.34
23.	86° 15′	5175	3431	22	3430.39
24.	90° 00′	5400	3438	7	3437.75

Note: The circumference of a circle, in arc minutes, is

$$2\pi R = 360^\circ = 21{,}600'$$

so that

$$R = 21600' / 2\pi = 3437.7468'$$

Example: Find the equations of the centre and hence the true longitudes of the Sun and the Moon at the mean midnight at Banglore, between March 21 and 22, 1991.

We have already computed the mean longitudes after the *deśāntara* correction, for the midnight of the given date at Banglore, and the values are:

Mean longitude of the Sun	:	334° 46′ 31″
Mean longitude of the Moon	:	48° 21′ 12″
Moon's *mandocca*	:	247° 57′ 20″
Sun's *mandocca*	:	77° 17′ 39″

Note: According to the **Sūryasiddhānta**, the Sun's apogee (*mandocca*) completes 387 revolutions in a *Kalpa* of 432×10^7 years. At that rate of motion, the position of the Sun's *mandocca* at the beginning of the *Kaliyuga* (i.e., February 17/18, 3102 BC) works out to be 77° 7′ 48″. Therefore, for the *ahargaṇa* of 18,59,872, corresponding to March 22, 1991, the motion of the Sun's *mandocca*

$$= [18{,}59{,}872 \,/\, (157{,}79{,}17{,}828 \times 10^3)] \times 387 \times 360 \times 60'$$
$$= 9.853' = 9'51''$$

Therefore, for the given date,

the Sun's *mandocca* = 77° 7′ 48″ + 9′ 51″ = 77° 17′ 39″

(i) The Sun's equation of the centre and true longitude:

the Sun's mean longitude	:	334° 46′ 31″
the Sun's *mandocca*	:	77° 17′ 39″

Therefore,

the Sun's anomaly (*mandakendra*) :

m = Sun's *mandocca* – Sun's mean longitude
= 77° 17′ 39″ – 334° 46′ 31″
= 102° 31′ 08″ (by adding 360°)
= 102.5189°

and hence

the rectified periphery $= 14^\circ - ({}^1/{}_3)^\circ\,|\sin m| = 13.6745^\circ$

The Sun's equation of the centre

$$= 133.7' \sin m - 3.183' (\sin m) \, |\sin m|$$
$$= (133.7')(0.9765377) - (3.183')(0.953626)$$
$$= 130.5631' - 3.0353917'$$
$$= 127.52771' = 2^\circ\, 7'\, 32''$$

Therefore, at the mean local midnight at Bangalore, the true longitude of the Sun

= (mean longitude of the Sun + equation of centre of the Sun)

$$= 334^\circ\, 46'\, 31'' + 2^\circ\, 7'\, 32'' = 336^\circ\, 54'\, 03''$$

(ii) The Moon's equation of centre and true longitude:

Moon's mean longitude : $47^\circ\, 44'\, 48''$
Moon's *mandocca* : $247^\circ\, 57'\, 34''$

Therefore, we have the Moon's anomaly (*mandakendra*),

m = Moon's *mandocca* – Moon's mean longitude

$$= 247^\circ\, 57'\, 34'' - 47^\circ\, 44'\, 48''$$
$$= 200^\circ\, 12'\, 46'' = 200.21278^\circ$$

Hence,

the corrected periphery of the epicycle

$$= 32^\circ - ({}^1/_3)^\circ |\sin m| = 31.8848^\circ$$

The Moon's equation of centre

$$= [305.6' (\sin m) - 3.183' (\sin \mathrm{m}).\ |\sin (\mathrm{m})|]$$
$$= (305.6')(-0.3455074) - (3.183)(-0.1193754)$$
$$= -105.58709' + 0.3799719'$$
$$= -105.58709' + 0.3799719'$$
$$= -105.20712' = -(1^\circ\, 45'\, 12'')$$

Therefore, at the mean local midnight at Bangalore,

True longitude of the Moon

= Mean longitude of the Moon + Equation of centre of the Moon

$$= 47^\circ\, 44'\, 48'' - 1^\circ\, 45'\, 12''$$
$$= 45^\circ\, 59'\, 36''$$

8.4 True Daily Motions of the Sun and the Moon

The *mean* daily motions of the Sun and the Moon, as given by the **Sūryasiddhānta**, are respectively $59'8''\,10^{iii}\,10.4^{iv}$ (or $59'.1361592$) and $790'34''52^{iii}3.8^{iv}$ (or $790'.5811287$).

But due to the non-uniform motion of the Sun along the ecliptic, the true daily motions of the Sun and of the Moon go on changing from day to day. The method to find the true daily motion from the mean motion is given in the **Sūryasiddhānta** as follows:

True daily motion
$= n \pm (n-n').P^\circ \times$ [sine difference at $(m-\propto)$] $/(360^\circ \times 225')$

where P° is the periphery of the epicycle of the Sun (or the Moon), m is the mean longitude and $\propto$ is the mean longitude of the apogee (*mandocca*) of the Sun (or the Moon), and "sine difference" is the tabulated difference in the sine table corresponding to the mean anomaly, namely $(m-\propto)$; n and n' are the mean daily motions of the Sun (or the Moon) and the corresponding apogee.

RATIONALE: Suppose L and L' are the true longitudes of the Sun (or Moon) on two consecutive days. Then, we have

$$L = m \pm P^\circ \times [\sin(m-\propto)] / 360^\circ \text{ and}$$
$$L' = (m+n) \pm (P^\circ / 360^\circ) \sin\{(m+n)-(\propto + n')\}$$

so that

True daily motion

$$= L' - L$$
$$= n \pm (P^\circ / 360^\circ)\,[\sin\{(m-\propto)+(n-n')\} - \sin(m-\propto)]$$
$$= n \pm [P^\circ \times (n-n') / (360^\circ \times 225')] \times \text{Tab. diff of sines at } (m-\propto)$$

In the case of the Sun, the daily motion of its *mandocca* (apogee) n' is negligible so that:

True daily motion of the Sun

$$= n \pm P^\circ \times n \times [\text{ tab. diff. of sines at } (m-\propto)] / (360^\circ \times 225')$$

Note: The correction to the mean daily motion n is *additive* if the anomaly (of the Sun or the Moon) is between 90° and 270° (i.e., 2nd and 3rd quadrants), and *subtractive* if the anomaly is between 270° and 90° (i.e., in 1st and 4th quadrants).

Example: Find the true daily motions of the Sun and the Moon on March 22, 1991.

(i) TRUE DAILY MOTION OF THE SUN

We shall use the following values in respect to the Sun as on March 22, 1991, at the preceding midnight:

Sun's mean daily motion	: $n = 59'8''$
Sun's mean longitude	: $334° 51' 30''$
Sun's apogee (*mandocca*)	: $77° 17' 39''$
Sun's mean anomaly	: $102° 26' 09''$
Tabular difference of sines	: $51'$

(corresponding to $180° - 102° 26' 09'' = 77° 33' 51''$, see Table 8.2, between serial numbers 20 and 21).

Corrected periphery of the Sun's epicycle : $P = 13°.6745$

Therefore, the correction to the Sun's daily motion must be

$$\begin{aligned} & P \times n \times \text{Tab. sin diff.} / (360 \times 225) \\ & = 13°.6745 \times 59'.13 \times 51 / (360 \times 225) \\ & = 0'.5091 = 0'31'' \end{aligned}$$

Since the Sun's anomaly, $102° 26' 09''$, lies between 90° and 270° (i.e., 2nd and 3rd quadrants), the correction is *additive*. Hence,

$$\begin{aligned} \text{the Sun's true daily motion} &= 59'08'' + 0'31'' \\ &= 59'39''. \end{aligned}$$

(ii) TRUE DAILY MOTION OF THE MOON

In the case of the Moon, the following are the values for March 22, 1991, at the preceding midnight:

Moon's mean daily motion	: $n = 790' 35''$
Mean daily motion of the Moon's apogee	: $n' = 6'.683$

Moon's mean longitude	: $47^\circ\ 44'\ 48''$
Moon's apogee (*mandocca*)	: $247^\circ\ 57'\ 34''$
Moon's mean anomaly	: $200^\circ\ 12'\ 46''$
Tabular sine diff. for $(200^\circ 12'46'' - 180^\circ)$	: $210'$
Moon's corrected periphery of the epicycle	: $P = 31^\circ.8848$

Therefore,

The correction to the Moon's daily motion

$= (n - n') \times P \times \text{Tab. sin diff.} / (360 \times 225).$

$= (790.583' - 6.683') \times 31.8848^\circ \times 210' / (360^\circ \times 225')$

$= 64.8' = 64'\ 48''$

Since the Moon's anomaly $200^\circ\ 12'\ 46''$ lies between 90° and 270°, the correction is *additive*. Hence,

$$\text{The Moon's true daily motion} = 790'35'' + 64'48'' = 855'23'' = 14^\circ 15'23''$$

8.5 *Bhujāntara* Correction

The true midnight of a place differs from the mean midnight by an amount of time called the "*equation of time*". The equation of time is caused by:

(i) the eccentricity of the Earth's orbit; and
(ii) the obliquity of the ecliptic with the celestial equator.

The correction to the longitude of a planet, due to the part of the equation of time caused by the eccentricity of the Earth's orbit, is called *bhujāntara*. The other correction caused by the obliquity of the ecliptic is called *Udayāntara*.

While all the Siddhāntic texts consider the *bhujāntara* correction, the other correction – *Udayāntara* – was first introduced by Śrīpati (about 1025 AD) and later followed by Bhāskara II and others.

We shall discuss the *bhujāntara* correction which is mentioned in the **Sūryasiddhānta**. The eccentricity of the Earth's orbit results in the equation of the centre of the Sun – which is converted into time at the rate of 15° per hour or 6° per *ghaṭikā*. This rate of conversion is

due to the fact that the Earth rotates about its axis at the rate of 360° in 24 hours (or 60 *ghaṭikās*). The resulting amount in time units is the equation of time caused by the eccentricity of the Earth's orbit. Thus,

equation of time (due to the eccentricity)
= [(equation of centre of the Sun) / 15] hours
= [(equation of centre of the Sun) / 6] *ghaṭikās*

Now, to get the *bhujāntara* correction for the Sun or the Moon or any other planet, the equation of time obtained above must be multiplied by the motion of the planet per hour or per *ghaṭikā*, as the case may be. That is,

bhujāntara correction for a planet
= [equation of time in hours] × [(daily motion) / 24]
= (equation of the centre of the Sun) / 15 × (daily motion) / 24
= (equation of the centre of the Sun) × (daily motion) / 360

where the factors in the numerator are in degrees.

If the time unit used is *ghaṭikā*, then,

bhujāntara correction
= (eqn. of time in *ghaṭikā*) × (daily motion / 60)
= [(eqn. of centre of the Sun in degrees) / 6]
× [(daily motion of the planet) / 60]
= [eqn. of centre of the Sun in degrees]
× [daily motion of the planet] / 360

where the daily motion of the planet is in degrees, then *bhujāntara* correction is also in degrees.

However, if the daily motion of the planet is in minutes of arc, then

bhujāntara correction in degrees
= (eqn. of centre of the Sun in degrees)
× (daily motion of the planet in minutes) / 21600

Further, the *bhujāntara* correction is additive or subtractive according to whether the equation of the centre of the Sun is similar or not.

For example, in the case of the Moon, its *mean* daily motion is $13.176352°$ or $790.58112'$. Therefore, we have,

bhujāntara correction (mean)
$= \text{(eqn. of centre of the Sun)} \times 790.58112' / 21600'$
$= \text{eqn. of centre of the Sun} / 27.321674$

Note: Brahmagupta takes the denominator approximately as 27 in his **Khaṇḍakhādyaka**.

It is important to note that to obtain the actual (and not the mean) *bhujāntara* correction of a planet, we have to use the true daily motion of the planet for the given day.

Example: Find the *bhujāntara* corrections for the longitudes of the Sun and the Moon on March 22, 1991.

True daily motion of the Sun	:	$59.65'$
True daily motion of the Moon	:	$855.23'$
Equation of the centre of the Sun	:	$+2° 7'32'' = 127.53'$

Therefore, we have,

(i) Actual *bhujāntara* correction of the Sun

$= \text{(eqn. of the centre of the Sun)} \times$
$\text{(daily motion of the Sun)} / 21600'$
$= 127.53' \times 59.65' / 21600'$
$= 0.3521835'$
$= 0'21''$.

Since the equation for the centre of the Sun is additive, the *bhujāntara* correction is also additive. Therefore,

True longitude of the Sun $= 336°59'02'' + 0'21''$
$= 336°59'23''$

(ii) Actual *bhujāntara* correction of the Moon

$= \text{(eqn. of centre of the Sun)}$
$\times \text{(daily motion of the Moon)} / 21600$
$= 127.53' \times 855.23' / 21600'$
$= 5.0494205' = 5'3''$

Here, also, the correction is additive since the Sun's equation of centre is so. Therefore,

$$\text{Longitude of the Moon} = 45°59'36'' + 5'03''$$
$$= 46°04'39''$$

Note: A **computer program,** "SSRAMOON", based on the **Sūryasiddhānta** is provided after Chapter 12.

8.6 Further Corrections for the Moon

So far we have applied an important correction, namely, the equation of the centre to the mean position of the Moon. Besides this correction, the other two corrections applied, viz., *deśāntara* and *bhujāntara* are mainly to determine the position of the Moon at true midnight at the place of observation.

However, to get the true apparent position of the Moon, two more important corrections will have to be applied, ignoring of course the other minor corrections due to planetary disturbances. These are:

(i) Evection $= (15/4)\, m\, e \sin(2\xi - \phi)$
$= 76'\, 26'' \sin(2\xi - \phi)$.

where m is the ratio of mean daily motion of the Sun to that of the Moon, e is the eccentricity of the Moon's orbit, $\xi = (M - S)$, the elongation of the Moon from the Sun and $\phi = M - A$, the mean anomaly of the Moon (A being the Moon's apogee).

(ii) Variation $= 39'30'' \sin(2\xi)$

In the above formulae, S and M are respectively the mean longitudes of the Sun and the Moon. The **Sūryasiddhānta**, being an earlier text, does not mention these corrections. However, Mañjula (932 AD), Bhāskara II (1150 AD) and later Indian astronomers have recognized the *evection* correction in addition to the equation of the centre.

Actually, the *evection* correction (combined with a part of the equation of centre) was first given, among the Indian astronomers, by Mañjula (or Muñjala, 932 AD) in his **Laghumānasam.** P.C. Sengupta

remarks, "In form the equation is most perfect, it is far superior to Ptolemy's, it is above all praise."

Apart from the above three major equations of the moon, there is another important fourth equation called the *annual equation*. The credit for the discovery of this lunar equation, among Indian astronomers, goes to the Oriyan astronomer, Candraśekhara Sāmanta (19th cent.). It is noteworthy that Candraśekhara discovered this important correction independently, since he was trained in the traditional and orthodox Sanskrit style and totally ignorant of English education or the western development of astronomy. Candraśekhara's equation is

$$\text{Annual correction} = (11'27''.6) \sin (\text{Sun's anomaly})$$

Tycho Brahe assumed the coefficient wrongly as $4'30''$ while Horrock's (1639 AD) value is $11'5''$.

9

True Positions of the Star-planets

9.1 *MANDA* CORRECTION FOR THE STAR-PLANETS

The mean positions of the planets, viz., *Kuja, Budha, Budha's śīghrocca, Guru, Śukra, Śukra's śīghrocca* and *Śani* were obtained, as explained in section 7.4, by assuming a uniform circular motion for them. Since by observations it was found that the motion of each planet is non-uniform, suitable corrections were devised. These are called *manda* and *śīghra* corrections.

The *manda* correction for the five planets is similar to that for the Sun and the Moon discussed in Chapter 8. The points S and S' in Fig. 8.1, now represent the true and the mean planet.

The *mandaphala* in the case of a planet is given by

$$\textit{Mandaphala} = (r/R)\,(R \sin m)$$

where $R \sin m$ is the Indian sine of the planet's anomaly m, r is the radius of the epicycle of "apsis" as distinguished from the epicycle of "conjunction" which will be discussed shortly. The radius r of the *manda* epicycle is a variable (see Table 9.1).

Table 9.1 Peripheries of *manda* epicycles

Planet	Periphery of *manda* epicycle: At the end of odd quadrants	Periphery of *manda* epicycle: At the end of even quadrants
Kuja	72°	75°
Budha	28°	30°
Guru	32°	33°
Śukra	11°	12°
Śani	48°	49°

The corrected periphery p for any given *manda* anomaly m is:

$$p = p_e - (p_e - p_o)\ |\sin m|$$

where p_e and p_o are, respectively, the peripheries of a planet at the ends of even quadrants (i.e., at $m = 180°$ and $m = 360°$ or $0°$) and at the ends of odd quadrants (at $m = 90°$ and $m = 270°$); m is the *manda* anomaly of the planet given by

manda anomaly = *mandocca* of the planet – mean planet

For example, in the case of *Kuja*, the corrected periphery of the *manda* epicycle is given by:

$$\begin{aligned} p &= 75° - (75° - 72°)\ |\sin m| \\ &= 75° - 3°\ |\sin m| \end{aligned}$$

The *mandoccas* of the five planets are given in Table 9.2 in terms of the revolutions completed in the course of a *Kalpa*.

The *mandoccas* at the beginning of the *Kaliyuga* are also provided in Table 9.2.

Table 9.2 Revolutions of mandoccas in a *Kalpa* and their positions at the beginning of *Kaliyuga*

Planet	**No. of revns. in a *Kalpa***	***Mandocca* at the beginning of *Kaliyuga***
Kuja	204	4s 09°57′36″
Budha	368	7s 10°19′12″
Guru	900	5s 21°00′00″
Śukra	535	2s 19°39′00″
Śani	39	7s 26°36′36″

The method of calculating the *mandocca* of a planet on a given day, with *ahargaṇa* A, using Table 9.2 is as follows:

Mandocca
= (*mandocca* at the beginning of *Kaliyuga*) + (no. of revns. in *Kalpa*) × 360° × A / (no. of civil days in a *Kalpa*)

Number of civil days in a *Kalpa* = 1577917828000

Examples:

(i) Find the *mandocca* of *Guru* as on March 22, 1991. We have, for the given day, $A = 1859872$.

According to Table 9.2, the number of revolutions of *Guru's mandocca* in a *Kalpa* is 900 and its position at the beginning of the *Kaliyuga* is 5s 21°. Therefore,

Guru's mandocca

$$= 5s\ 21^\circ + [900 \times 360^\circ \times 1859872 \ / \ 1577917828000]$$
$$= 5s\ 21^\circ + 0^\circ 23' = 5s\ 21^\circ\ 23'$$

(ii) Find the *manda* correction for Jupiter (*Guru*) as on March 22, 1991 at the preceding midnight.

Guru's manda anomaly,

$$\begin{aligned} m &= \textit{Guru's mandocca} - \text{mean longitude of } \textit{Guru} \\ &= (5s\ 21^\circ\ 23') - (3s\ 18^\circ\ 29') \\ &= 2s\ 2^\circ\ 54' \\ &= 62^\circ\ 54' = 62.9^\circ \end{aligned}$$

Guru's corrected periphery of the *manda* epicycle,

$$\begin{aligned} p &= p_e - (p_e - p_o)\ |\sin \mathrm{m}| \\ &= 33^\circ - (33^\circ - 32^\circ)\ |\sin(62.9^\circ)| \\ &= 33^\circ - 1^\circ |\sin(62.9^\circ)| \\ &= 32.109787^\circ = 32^\circ 6' 35'' \end{aligned}$$

Therefore, we have

Guru's mandaphala

$$\begin{aligned} &= 3438' \times (p^\circ / 360^\circ) \times \sin m \\ &= 3438' \times (32.109787^\circ / 360^\circ) \times \sin(62.9^\circ) \\ &= 272.98239' \\ &= 4^\circ\ 32'\ 59'' \end{aligned}$$

Since *Guru's* anomaly m is less than 180°, the *mandaphala* is additive.

Note: In the **Sūryasiddhānta**, the anomaly m of a planet is defined by

$$m = \textit{Mandocca} - \text{mean planet}$$

and the corresponding *mandaphala* is additive for $m < 180^\circ$.

But, generally, in other Siddhāntic texts, m is defined as mean planet *minus* its *mandocca*. Accordingly, the resulting *mandaphala* is subtractive and additive, respectively, for $m < 180°$ and $m > 180°$. However, the result is the same from both the methods in correcting the mean position of a planet by the *manda* equation.

9.2 *ŚIGHRA* CORRECTION FOR THE STAR-PLANETS

The *śīghra* correction corresponds to the "elongation" in the case of *Budha* and *Śukra* from the Sun, and the annual parallax in the case of *Kuja, Guru* and *Śani.*

The *manda* correction is applied to the mean longitude of a planet to get the "true-mean" position of the planet (*mandasphuṭa graha*).

Now, the concept of the *śīghra* correction is explained with the help of Fig.9.1.

Let the circle *CDFG*, with the Earth at the centre *E*, represent the *kakṣyavṛtta* (or deferent circle) of a planet. Just like the *manda* epicycle, a *śīghra* epicycle is prescribed with a specified variable radius for each planet. Let *C* be the centre of the *śīghra* epicycle of the planet. While *C* moves along the deferent circle, the planet moves along its epicycle. The epicycle in this case is called *śīghra-nīcocca-vṛtta*. Let *CEF* cut the

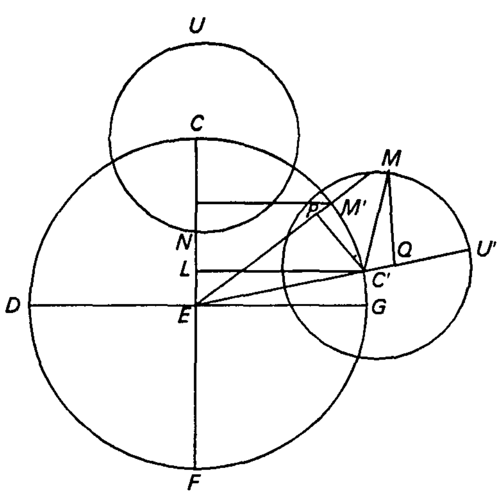

Figure 9.1 *Śighra* epicycle

epicycle at U and N which are, respectively, *śīghrocca* and *śīghranica* (*śīghra* apogee and perigee) of the *śīghra* epicycle. The centre C of the epicycle moves along the deferent circle with the velocity of the corrected planet (*mandasphuṭa graha*). Let the planet move from U' to M along the epicycle so that arc $U'M$ is equal to arc $C'C$. Join EM, cutting the deferent at M'. Then C' is the *mandasphuṭa graha* and M' is the true planet (*sphuṭagraha*).

Therefore, the correction to be made to the longitude of the "true-mean" planet (i.e., *manda*-corrected-planet) is the arc $C'M'$. The correction arc $C'M'$ in angular measure, is called *śīghraphala*. Now, in order to obtain an expression for the arc $C'M'$, draw $C'L$, $C'P$ and MQ perpendiculars, respectively, to CE, EM and $U'E$.

The angle $C'EC$, which is the angle between the *śīghrocca* and the *mandasphuṭagraha*, is called the *śīghra kendra* or the anomaly of conjunction.

From Fig. 9.1, we have

$$C'L = R \sin (\textit{śīghra}\text{ anomaly})$$
$$EL = R \cos (\textit{śīghra}\text{ anomaly})$$

Also, arc $U'M$ = arc $C'C$ and angle $U'C'M$ = angle $C'EC$, and hence the triangles $MC'Q$ and $C'EL$ are similar. Therefore,

$$\begin{aligned} MQ / MC' &= C'L / C'E \\ MQ &= C'L \times MC' / C'E \\ &= R \sin (\textit{śīghra}\text{ anomaly}) \times \text{Radius of epicycle} / R \\ &= R \sin (\textit{śīghra}\text{ anomaly}) \times \text{Epicylic periphery} / 360^\circ \\ &= \textit{dohphala} \end{aligned}$$

Again, from the above similar right-angled triangles, we have

$$\begin{aligned} C'Q / C'M &= EL / EC' \\ C'Q &= EL \times MC' / C'E \\ &= R \cos (\textit{śīghra}\text{ anomaly}) \times \text{Radius of epicycle} / R \\ &= R \cos (\textit{śīghra}\text{ anomaly}) \times \text{Epicyclic periphery} / 360^\circ \\ &= \textit{koṭiphala} \end{aligned}$$

Now, *sphuṭakoṭi*

$$EQ = EC' + C'Q = R + \textit{koṭiphala}$$

The *koṭiphala* is positive or negative according to whether the *śīghra* anomaly is in the fourth and first quadrants (i.e. between 270° and 90°), or in the second and third quadrants (i.e. from 90° to 270°).

Then, we have *śīghrakarṇa*,

$$EM = \sqrt{EQ^2 + MQ^2} = \sqrt{(C'E + C'Q)^2 + MQ^2)}$$

the hypotenuse of the right-angled triangle MEQ. From the similar triangles $EC'P$ and EMQ, we have

$$C'P / C'E = MQ / EM$$
$$C'P = MQ \times EC' / EM$$
$$= (Dohphala \times R) / Śīghrakarṇa$$

Then, the *śīghraphala* arc $C'M'$ is the arc corresponding to $C'P$ as R sine (*śīghra* anomaly). It is important to note that in the **Sūrya-siddhānta**, we have:

Śīghra anomaly = *śīghrocca* – mean planet

In the case of the superior planets, viz., *Kuja, Guru* and *Śani,* their *śīghrocca* is the same as the mean longitude of the Sun.

In the case of *Budha* and *Śukra*, their mean longitude is taken to be that of the Sun while their *śīghrocca*s are special points. In the Siddhāntic texts while the revolutions of the other mean planets, in a *Kalpa* or a *Mahāyuga*, are given, in the case of *Budha* and *Śukra*, the revolutions of their *śīghroccas* are given. Thus, we have, according to the *Sūryasiddhānta*:

(i) For the superior planets, viz., *Kuja, Guru* and *Śani,*

śīghra anomaly = mean Sun – mean planet

(ii) For the inferior planets, viz., *Budha* and *Śukra*,

śīghra anomaly = the planet's *śīghrocca* – mean Sun

In both the cases, we have:

R sin (*śīghraphala*) = (r / k) $(R \sin m)$

where r is the corrected radius of the *śīghra* epicycle of the planet, k is the *śīghra* hypotenuse (*śīghrakarṇa*) and $R \sin m$ is the Indian sine of the *śīghra* anomaly m of the planet. It is important to note

that the radius *r* of the *śīghra* epicycle is a variable as in the case of the *manda* epicycle.

The peripheries of the *śīghra* epicycles of the five star-planets are listed in Table 9.3.

Table 9.3 Peripheries of *śīghra* epicycles

	Periphery of *śīghra* epicycle	
Planet	**At the end of odd quadrants**	**At the end of even quadrants**
Kuja	232°	235°
Budha	132°	133°
Guru	72°	70°
Śukra	260°	262°
Śani	40°	39°

As in the case of the *manda* epicycle, the radius and the periphery of the *śīghra* epicycle are variable. The corrected periphery *p* for a given *śīghra* anomaly is given by

$$p = p_e - (p_e - p_0)\ |\sin m|$$

where p_e is the periphery of a planet's epicycle at the end of an even quadrant, p_0 is at the end of an odd quadrant and *m* is the *śīghra* anomaly. The *śīghra* anomaly, as pointed out earlier, is given by

śīghra anomaly = *śīghrocca* – mean planet

Example: Find the *śīghra* anomaly and, thus, the corrected periphery of the *śīghra* epicycle in the case of *Budha* and *Śani* for the midnight preceding March 22, 1991, at Bangalore.

For the given date, time and place, we have found out the mean positions (after the *deśāntara* correction) as given below:

Mean longitude of the Sun : 334°51′30″
Mean *śīghrocca* of *Budha* : 67°46′46″
Mean longitude of *Śani* : 272°49′04″

(i) Therefore, we have:

Budha's śīghra anomaly

$$m = 67^\circ\, 46'\, 46'' - 334^\circ\, 51'\, 30''$$
$$= 92^\circ\, 55'\, 16''\ \text{(after adding } 360^\circ) = 92.921^\circ$$

The corrected periphery of *Budha's śīghra* epicycle is:

$$\begin{aligned} p &= p_e - (p_e - p_o)\ |\sin m| \\ &= 133^\circ - (133^\circ - 132^\circ)\ (0.9987) \\ &= 132.0013^\circ \text{ or } 132^\circ\ 0'\ 4''.7 \end{aligned}$$

(ii) In the case of *Śani*, we have its *śīghra* anomaly,

$$\begin{aligned} m &= \textit{śīghrocca} \text{ of } \textit{Śani} - \text{mean } \textit{Śani} \\ &= \text{mean Sun} - \text{mean } \textit{Śani} \\ &= 334^\circ 51' 30'' - 272^\circ 49' 04'' \\ &= 62^\circ 02' 26'' = \ 62.0406^\circ \end{aligned}$$

The corrected periphery of *Śani's śīghra* epicycle p is,

$$\begin{aligned} p &= 39^\circ - (39^\circ - 40^\circ)\ |\sin 62.0406^\circ| \\ &= 39^\circ + (1^\circ)\ (0.88328) \\ &= 39.88328^\circ = 39^\circ 53' \end{aligned}$$

The *śīghra* anomaly and the corrected *śīghra* periphery for the remaining planets can also be calculated.

9.3 Working Rule to Determine the *Śīghra* Correction

After finding the *śīghra* anomaly m and the corrected periphery p of the *śīghra* epicycle for a planet, the *śīghra* correction is determined as follows (with $R = 3438$):

(i) *Dohphala* $= (p^\circ / 360^\circ) \times R \sin (m)$

(ii) *Koṭiphala* $= (p^\circ / 360^\circ) \times R \cos (m)$

(iii) *Sphuṭakoṭi* $= R \pm$ *koṭiphala*
where the positive or the negative sign is taken according to whether m lies between 270° and 90° or between 90° and 270°.

(iv) *Śīghrakarṇa* (or *śīghra* hypotenuse)

$$= \sqrt{(\textit{sphuṭakoṭi})^2 + (\textit{dohphala})^2}$$

(v) Then, we have

$$R \sin (\textit{śīghraphala}) = (\textit{dohphala} / \textit{śīghrakarṇa}) \times R$$
and $\textit{śīghraphala} = \sin^{-1} (\,|\textit{dohphala} / \textit{śīghrakarṇa}|\,)$

(vi) The *śīghraphala* is additive or subtractive according to whether the *śīghra* anomaly is less than 180° or greater than 180°.

From the above working rule, we have:

$$Sphu\d{t}ako\d{t}i = R \pm (p/360)\, R \cos m$$
$$= R\,[1 \pm (p/360) \cos m]$$

$$Dohphala = (p/360)\, R \sin m$$

Śīghrakarṇa

$$= \sqrt{(p^2 / 360^2)R^2(\sin^2 m) + R^2[1 \pm (p / 360) \cos m]^2}$$
$$= R\sqrt{(p^2 / 360^2)(\sin^2 m) + (p^2 / 360^2)(\cos^2 m) \pm 2(p / 360)(\cos m) + 1}$$
$$= R\sqrt{(p^2 / 360^2) \pm 2(p / 360)(\cos m) + 1}$$

Therefore, we have

Sīghraphala

$$= \sin^{-1}\left[[(p / 360)(\sin m)] / \sqrt{(p^2 / 360^2) \pm 2(p / 360)(\cos m) + 1}\right]$$
$$= \sin^{-1}\left[r \sin m / \sqrt{r^2 \pm 2r \cos m + 1}\right]$$

where $r = (p / 360°)$ is the corrected radius of the epicycle.

In the above formula, for the alternative $\pm$, the positive sign should be used if m is greater than 270° but less than 90° (i.e., $270° < m < 360°$ or $0° < m < 90°$); and the negative sign if $90° < m < 270°$.

Example: Find the *śīghra* correction for *Budha* and *Śani* at the midnight preceding March 22, 1991, at Bangalore.

As calculated earlier, we have the following values for the *śīghra* anomaly and the corrected epicycle:

Budha's corrected periphery, $p = 132.0013°$
Budha's *śīghra* anomaly, $m = 92.921°$
Śani's corrected periphery, $p = 39.88328°$
Śani's *śīghra* anomaly, $m = 62.0406°$

(1) *ŚĪGHRA* CORRECTION FOR *BUDHA*:

(i) *Dohphala* $= (132.0013 / 360) \times 3438' \times \sin(92.921°)$
$= 1258'.97456$

(ii) *Koṭiphala* = $(132.0013 / 360) \times 3438' \times \cos(92.921°)$
$= -64.23953'$

(iii) *Sphuṭakoṭi* = $3438' - 64.717731' = 3373.76047'$

(iv) *Sīghrakarṇa* = $\sqrt{(3373.76047')^2 + 1258.97456')^2}$
$= 3601.0105'$

(v) $R \sin$ (*sīghraphala*) = $1258.97456' \times 3438' / 3601.0105'$
$= 1201.9833'$

Therefore,

Sīghraphala = $\sin^{-1}[1201.9833 / 3438]$
$= 20.463892°$
$= 20° 27' 50''$

Since the *śīghra* anomaly, $m = 92.921°$ is less than 180°, the *śīghra* correction is additive,

i.e., *śīghra* correction $= +20°27'22''$

(2) *ŚĪGHRA* CORRECTION FOR *ŚANI*:

(i) *Dohphala* = $(39.88328 / 360) \times 3438' \times \sin(62.0406°)$
$= 336.4284'$

(ii) *Koṭiphala* = $(39.88328 / 360) \times 3438' \times \cos(62.0406°)$
$= 178.57648'$

(iii) *Sphuṭakoṭi* = $3438' + 178.57648' = 3616.5765'$

(iv) *Sīghrakarṇa* = $\sqrt{(33.4284')^2 + (3616.5765')^2}$
$= 3632.1907'$

(v) $R \sin$ (*Sīghraphala*) = $336.4284' \times 3438' / 3632.1907'$
$= 318.44166'$

Therefore,

Sīghraphala = $\sin^{-1}[318.44166 / 3438]$
$= 5.3145878°$
$= 5°18'53''$

Since the *śīghra* anomaly, $m = 62.0406°$ is less than 180°, the correction is additive.

Śīghra correction $= +5°18'53''$.

9.4 Application of *Manda* and *Śīghra* Corrections to the Star-planets

In the case of the five star-planets, viz., *Budha, Śukra, Kuja, Guru* and *Śani*, the *manda* and *śīghra* corrections are applied successively one after the other, according to the prescribed rules, to get the true position of the planet. In fact, though the prescribed rules slightly differ from text to text, essentially the application is an iterative process for getting a convergent value as the true position.

The **Sūryasiddhānta** gives the following procedure to apply the *manda* and the *śīghra* corrections successively.

1st Operation:

To the mean planet add half of the *śīghra* correction. Let MP be the mean longitude of the planet (after *deśāntara* correction) and SE_1 be the *śīghra* correction calculated for MP. Then, the position of the planet after the first operation is given by

$$P_1 = MP + (1/2)\, SE_1$$

2nd Operation:

To the position thus obtained from the 1st operation, add half of the corresponding *manda* correction.

For the first corrected position P_1, assume the corresponding *manda* correction to be ME_1. Then, the position of the planet after this second correction is given by

$$P_2 = P_1 + (1/2)\, ME_1$$

3rd Operation:

From the position thus corrected, find the *manda* correction and apply it *entirely* to the *original* mean position of the planet.

Thus, the *manda* correction ME_2 is determined corresponding to the twice-corrected position of the planet, namely P_2 and then it is applied to the original mean position MP of the planet. That is, the position after the 3rd operation is given by

$$P_3 = MP + ME_2$$

where the *manda* correction ME_2 is calculated taking P_2 as the mean-planet.

Finally, the 4th operation is effected to get the true position of the planet.

4th Operation:

From the position of the planet obtained after the 3rd operation, find the *śīghra* correction and apply the *whole* of it to the above.

This means that the *śīghra* correction SE_2, obtained from the position P_3 of the planet, is applied entirely to P_3. The position P_4 after this 4th correction is, therefore, given by

$$P_4 = P_3 + SE_2$$

Note: (1) The above four operations may be repeated in the same order, treating P_4 as the new mean position of the planet. A repeated application of this cycle of four operations refines the position of the planet, iteratively by successive approximations. (2) A computer program, "SSPLA" is provided after chapter 12.

Example: Find the true position of *Śani*, by applying the four operations of the *manda* and *śīghra* corrections, for the midnight preceding March 22, 1991, at Bangalore.

We have already calculated the following:

Mean longitude of *Śani* (*MP*) (after *deśāntara* correction)	:	272° 49′ 4″
Mean longitude of the Sun	:	334° 51′ 30″
Corrected periphery of *Śani's* *śīghra* epicycle	:	39.88328°
Śīghra anomaly of *Śani*	:	62.0406°.

1st Operation:

We found in section 9.3 that for the mean position *MP* of *Śani*, the

$$\text{Śīghra equation} = +5°18'53'' = SE_1$$

Therefore, after the 1st correction, *Śani's* position P_1 is :

$$P_1 = MP + (\tfrac{1}{2})\, SE_1$$
$$= 272°49'04'' + 2°39'27'' = 275°28'31''$$

2nd Operation:

The position of *Śani* after the 2nd operation, P_2, is given by

$$P_2 = P_1 + (½)\ ME_1$$

where ME_1 is the *manda* correction corresponding to the first corrected position P_1 of *Śani.*

We have for the given day, the *Kali ahargaṇa* $A = 1859872$. Using Table 9.2,

Śani's mandocca
$= (7s\ 26°36'36'') + (39 \times 360° \times 1859872 / 1577917828000)$
$= 7s\ 26°37'36''$

Therefore, *Śani's manda* anomaly,

$m =$ *Śani's mandocca* – mean longitude of *Śani*
$= 7s\ 26°37'36'' - 9s\ 24°90'4''$
$= 10s\ 23°48'32''$
$= 323.8089°$

Śani's corrected periphery of the *manda* epicycle,

$p = 49° - (49° - 48°)\ |\sin 323.8089°|$
$= 48.40952°$

Therefore,

Śani's mandaphala
$= 3438' \times (48.40952° / 360|) \times \sin (323.8089°)$
$= -272.98549°$

i.e., $ME_1 = -4°32'59''$

It is negative since $m > 180°$.

Therefore, the position of *Śani* after this second operation is given by

$P_2 = P_1 + (½)\ ME_1$
$= 275°28'31'' - (½)\ (4°32'59'')$
$= 275°28'31'' - 2°16'30''$
$= 273°12'01''$

3rd Operation:

The position after the 3rd operation is given by

$$P_3 = MP + ME_2$$

where $MP = 272°49'04''$, the mean longitude of *Śani*, and ME_2 is the *manda* correction obtained for the last corrected position P_2.

Now, *Śani's* new *manda* anomaly,

$$\begin{aligned} m &= \textit{Śani's mandocca} - P_2 \\ &= 236°37'36'' - 273°12'01'' \\ &= 323°25'35'' = 323.426389° \end{aligned}$$

Śani's corrected periphery of the *manda* epicycle:

$$\begin{aligned} p &= 49° - 1° \times |\sin(323.426389°)| \\ &= 48.404145° \end{aligned}$$

Therefore, *Śani's mandaphala* (new):

$$\begin{aligned} ME_2 &= (3438' \times 48.40145° / 360°) \times \sin(323.426389°) \\ &= -275.4397' = -4°35'26'' \end{aligned}$$

The position of *Śani* after this third operation is, therefore, given by

$$\begin{aligned} P_3 &= MP + ME_2 \\ &= 272°49'04'' - 4°35'26'' \\ &= 268°13'38'' \end{aligned}$$

4th Operation:

Finally, in the last correction SE_2, the second *śīghra* correction is determined for the last corrected position, i.e., P_3 and applied to the same. Thus, the 4th corrected position P_4 is given by,

$$P_4 = P_3 + SE_2$$

The new *śīghra* anomaly of *Śani*,

$$\begin{aligned} m &= \textit{śīghrocca} \text{ of } \textit{Śani} - P_3 \\ &= 334°51'30'' - 268°13'38'' \\ &= 66°37'52'' \\ &= 66.63111° \end{aligned}$$

The corrected periphery of *Śani's śīghra* epicycle p is given by,

$$p = 39^\circ + 1^\circ \times |\sin 66.63111^\circ|$$
$$= 39.9179701^\circ \text{ or } 39^\circ 55' 05''$$

Next, we shall calculate the *dohphala, koṭiphala, sphuṭakoṭi* and *sphuṭakarṇa* to obtain the resulting *sīghraphala.*

(i) $Dohphala = (p^\circ / 360^\circ) \times R \times \sin(m)$
$= (39.9179701 / 360) \times 3438' \times \sin(66.63111^\circ)$
$= 349.94546'$

(ii) $Koṭiphala = (p^\circ / 360^\circ) \times R \times \cos(m)$
$= (39.9179701 / 360) \times 3438' \times \cos(66.63111^\circ)$
$= 151.20939'$

(iii) $Sphuṭakoṭi = R + koṭiphala = 3589.2094$

(iv) $Sīghrakarṇa = \sqrt{(Sphuṭakoṭi)^2 + (Dohphala)^2}$
$= \sqrt{(3589.2094)^2 + (349.94546)^2}$
$= \sqrt{12882424.12 + 122461.82}$
$= 3606'.2288$

(v) $Śīghraphala = \sin^{-1}(|dohphala / sīghrakarṇa|)$
$= \sin^{-1}(|349.94546 / 3606.2288|)$
$= 5.5686983^\circ$
$= +5^\circ 34' 07'' = SE_2$

Since the *śīghra* anomaly is $m = 66.63111^\circ < 180^\circ$, the *śīghra* correction is additive.

Therefore, the finally corrected true longitude of *Śani* is:

$$P_4 = P_3 + SE_2$$
$$= 268^\circ 13' 38'' + 5^\circ 34' 07'' = 273^\circ 47' 45''.$$

9.5 True Daily Motions of Star-planets

The mean daily motions of the five star-planets, viz., *Kuja, Budha's śīghrocca, Guru, Śukra's śīghrocca* and *Śani,* are obtained assuming their uniform motion. However, due to the *manda* and *śīghra* corrections, the motions are not uniform. Therefore, to get

the *true* daily motion of a star-planet, we have to apply the following corrections.

According to the **Sūryasiddhānta**, the following procedure is prescribed for obtaining the true daily motion:

(1) To the *mean* daily motion n of the planet, apply the correction due to *manda*, which is similar to the one applied in the case of the Sun and the Moon (see section 8.4).

For this purpose, from the *third operation* in the process of obtaining the true position, the planet's longitude P_2 and the equated *manda* anomaly may be used. From this, we get the planet's *manda*-corrected daily motion n_1.

(2) From the planet's *śīghrocca* mean daily motion n_2, subtract the planet's *manda*-corrected daily motion n_1 (obtained from the previous step). This gives the planet's equated daily synodical motion $(n_2 - n_1)$. Note that in the case of *Kuja, Guru* and *Śani,* their *śīghrocca* is the mean longitude of the Sun.

(3) Let k be the *sīghrakarṇa* (hypotenuse) of the planet used in the last operation for finding the true position.

The *excess* of the *sīghrakarna* over the radius of the deferent circle is given by

excess = (*sīghrakarṇa* – radius) = (*sīghrakarna* – 3438′)

where *sīghrakarṇa* is in minutes of the arc, i.e., excess = $(K–R)$.

Then, the correction due to *śīghra* to n for getting the *true* daily motion of the planet is ascertained by

śīghra correction = (Excess / *sīghrakarṇa*) × (Equated syn. motion)

$$= (k - R)(n_2 - n_1) \; / \; k$$

The *śīghra* correction, thus obtained, is *additive* or *subtractive* according to whether the *śīghra* anomaly of the planet is *less than* or *greater than* 180°.

The *śīghra* correction, thus obtained, is applied to the planet's *equated* daily motion obtained earlier; that is,

$$\text{true daily motion} = n_1 + [(k - R)(n_2 - n_1) \; / \; k].$$

Example: Find the true daily motion of *Śani* at the midnight preceding March 22, 1991.

The mean daily motion of *Śani*	: 0.0334393° i.e., $n = 0°2'0''$
Mean longitude of *Śani* (after *deśāntara* cor.)	: 272° 49′ 04″
Mean longitude of *śīghrocca* (i.e., the mean Sun)	: 334°51′30″
Mandocca of *Śani*	: 236°37′36″

In the *third operation* for ascertaining the true position of *Śani*, we had obtained:

Śani's equated longitude (P_2) : 273° 12′ 01″.

Therefore, *Śani*'s equated *manda* anomaly,

$$m = mandocca - P_2$$
$$= 236°37'36'' - 273°12'01'' = 323°\ 25'\ 35''$$

Tabulated difference of sines : 183′ (for 360° – 323° 25′ 35″ = 36° 34′ 25″, see Table 8.2 between Sl. nos. 9 and 10).

The corrected periphery of *Śani's manda* epicycle is given by

$$p = p_e - (p_e - p_o)\ |\sin m|$$
$$= 49° - 1°\ |\sin(323°25'35'')| = 48.404145°$$

Therefore, we have the correction to *Śani*'s motion (due to *manda*)

$$= n' \times p° \times \text{Tab. sin diff.} / (360° \times 225')$$
$$= 2' \times 48.404145° \times 183' / (360° \times 225')$$
$$= 0.218715'$$
$$= 13''$$

Since *Śani's manda* anomaly, $m = 323°25'35''$, lies in the 4th quadrant, the *manda* correction to the daily motion is *subtractive*; i.e., the correction to the daily motion due to *manda* is –13″.

Now, the mean daily motion of *Śani*, $n = 0°\ 2'\ 0''$	
the *manda* correction is	: –13″
Hence, *Sani's manda*-corrected daily motion n_1	: 0° 1′47″

Now, the mean daily motion of *śīghrocca* n_2
(i.e., daily motion of the Sun) : 59′08″
Deduct *Śani's manda*-corrected motion : −1′47″

Therefore, *Śani*'s equated daily synodical
motion $(n_2 - n_1)$: 57′ 21″

The variable hypotenuse used in the last process for finding the true place of *Śani*, $k = 3606.2288'$.

Its *excess* over the deferent radius $(=3438')$ is:

$$k - R = 3606.2288' - 33438 = 168.2288'$$

Therefore, the equation of motion due to *śīghra* is given by

$$\begin{aligned} \textit{śīghra}\text{ cor.} &= (\text{excess} \,/\, \textit{sīghrakarṇa}) \times (\text{equated syn. motion}) \\ &= (k-R)\,(n_2 - n_1) \,/\, k \\ &= (168.2288' \,/\, 3606.2288') \times 57.35' \\ &= 2.6753493' = 2'\ 40'' \end{aligned}$$

Since the variable hypotenuse is greater than the radius, the correction is *additive*, i.e., *śīghra* correction = +2′40″. Hence,

Śani's equated daily motion
(i.e., *manda* correction) : 1′47″
Śīghra correction : +2′ 40″
Hence, *Śani*'s true daily motion : 4′27″

Note: If n_1 is the mean daily motion of the planet after the *manda* correction, n_2 is the mean daily motion of the planet's *śīghra*, k is the hypotenuse (*sīghrakarṇa*) in the last operation of finding the planet's true position and $R = 3438'$ is the constant radius of the deferent circle, then,

$$\text{true daily motion} = [n_1 + (n_2 - n_1)\,(k-R) \,/\, k]$$

which, on simplification, can also be written as:

$$\text{True daily motion} = [n_2 - (n_2 - n_1)R \,/\, k]$$

In the case of *Budha* and *Śukra*, the Sun is their mean position and their *śīghroccas* are separately obtained along with the mean

positions of the other planets. Therefore, in the above formula, n_2 is the *śīghrocca's* daily motion and n_1 is the *manda*-corrected daily motion of the concerned planet (*Budha* or *Śukra*), which is the same as that of the Sun.

9.6 Retrograde Motion of Star-planets

The star-planets move from west to east, relative to the fixed stars, as seen from the Earth due to their natural motion. However, during certain periods each of these planets *appears* to move backwards, i.e., from east to west. Their celestial longitudes keep on decreasing instead of increasing, day by day, for some time. This apparent backward motion is called *vakra gati* (retrograde motion).

The phenomenon of retrograde motion is caused by the difference in the velocities of the Earth and the planet, i.e., the relative velocity. This phenomenon is demonstrated in Fig.9.2.

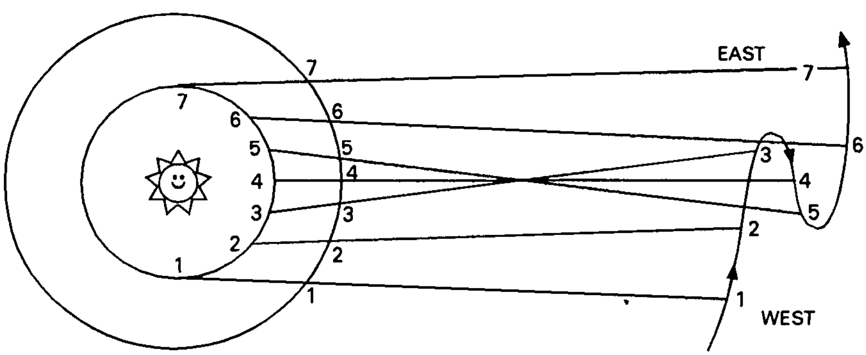

Figure 9.2 Retrograde motion of *Kuja*

In Fig. 9.2, the motion of Mars (*Kuja*) relative to the Earth is shown in the heliocentric model. The Earth's linear speed is 18.5 miles per second while that of Mars is 3.5 miles less, i.e., 15 miles per second. As the Earth overtakes Mars, the latter appears to move backwards, when seen from the Earth. The *direct* motion of Mars eastward is shown at positions 1, 2 and 3, *retrograde* at 4 and 5 westward and again *direct* motion eastward at 6 and 7.

The rule for determining the retrograde motion of a planet is given in **Sūryasiddhānta** as follows:

The retrograde motion (*vakra gati*) of the different star-planets commences when the *śīghrakendra* (i.e. *śīghra* anomaly), in the fourth process of determining true positions, is as follows:

Kuja	164°
Budha	144°
Guru	130°
Śukra	163°
Śani	115°

That is, the retrograde motion of *Kuja*, for example, commences when the *śīghrocca* (i e., Sun) – *Kuja* = 164°.

The point at which the motion of a planet changes from *direct* to *retrograde* is called a "*stationary point*". The planet remains retrograde for some days and then, again, its motion changes from retrograde to direct. This point of change is the second *stationary point.* At both the stationary points the planet has no apparent motion (i.e., the relative velocity is zero).

9.7 Rationale for the Stationary Point

In Fig. 9.3, let M be the mean planet, P be the true planet on the epicycle of radius p, E be the Earth and S the Sun. If n is the mean daily motion of the Sun, let t be the number of days since the Sun S was at the first point of *Meṣa*, $P\hat{M}K = \theta$ and $P\hat{E}M = E$, then the celestial longitude L of the planet is given by

$$L = nt - \theta + E$$

where nt is the longitude of the Sun. Therefore,

$$dL / dt = n - d\theta / dt + dE / dt \qquad \ldots \qquad (1)$$

Let $PM = p$ and $EM = r$, where the radii p and r are constants. In Fig. 9.3, we have $MA = p \cos\theta$ and $PA = p \sin\theta$. Therefore,

$$EA = EM + MA = r + p\cos\theta$$

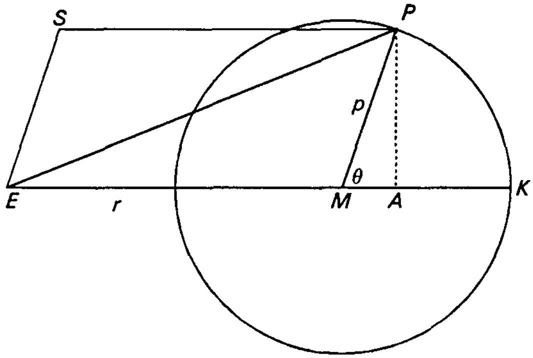

Figure 9.3 Stationary points

and hence

$$\tan E = PA \,/\, EA = p \sin\theta \,/\, (r + p\cos\theta)$$

so that

$$E = \tan^{-1}[p\sin\theta \,/\, (r + p\cos\theta)]$$

Differentiating this expression with respect to t we get,

$$dE \,/\, dt = (d\theta \,/\, dt)\,[p^2 + r\,p\cos\theta] \,/\, [r^2 + p^2 + 2r\,p\cos\theta] \quad \ldots \quad (2)$$

Substituting (2) in (1), we get

$$dL \,/\, dt = n - (d\theta \,/\, dt)\,[(r^2 + r\,p\cos\theta) \,/\, (r^2 + p^2 + 2r\,p\cos\theta)] \quad \ldots \quad (3)$$

If n is the mean daily motion of the Sun and n' is that of the planet and $\propto$ is a suitable constant, then

$$\theta = (n - n')\,(t + \propto)$$

so that

$$d\theta \,/\, dt = (n - n') \quad \ldots \quad (4)$$

Substituting (4) in (3), we get

$$dL \,/\, dt = [np^2 + n'r^2 + rp\,(n + n')\cos\theta] \,/\, [r^2 + p^2 + 2rp\cos\theta]$$

At the *stationary point* where the retrograde motion begins,

$$dL / dt = 0$$

Therefore,

$$n p^2 + n' r^2 + r p (n + n') \cos \theta = 0$$

so that,

$$\cos \theta = -(np^2 + n' r^2) / p r (n + n')$$

For example, in the case of *Kuja*, considering the mean values, we have $n = 0.98560265°$, $n' = 0.5240193°$, $p = 233.5°$, $r = 360°$. Here, p and r are taken as peripheries of the planet's *śīghra* epicycle and of the mean orbit, which are proportional to their radii. Substituting these values, we arrive at

$$\begin{aligned}\cos \theta &= \frac{-[0.98560265\,(233.5)^2 + 0.5240193\,(360)^2]}{[(233.5)\,(360)\,(1.5096219)]} \\ &= -[53737.271 + 67912.901] / [126898.82] \\ &= -121650.17 / 126898.82 = -0.9586391\end{aligned}$$

Therefore,

$$\theta = 180° - \cos^{-1}(0.9586391) = 163.4636°$$

The **Sūryasiddhānta** has taken this as 164°.

The other stationary point is given by $360° - \theta$, noting that

$$\cos \theta = \cos (360° - \theta)$$

In the above example, since $\theta = 164°$ according to the **Sūrya-siddhānta**, the second stationary point is $360° - 164° = 196°$. This means that *Kuja* will be retrograde during the period when its *śīghrakendra* (or *śīghra* anomaly) lies between 164° and 196°. Similarly, the corresponding limits for other planets can be calculated.

REMARK:

According to modern astronomy, the stationary value of the angle θ is given by

$$\cos \theta = -[a^{1/2}\, b^{1/2}] / [\, a - a^{1/2}\, b^{1/2} + b]$$

where a is the mean distance of the planet from the Sun, and b is the mean distance of the Earth from the Sun. Now, taking b as one astronomical unit, we have

$$\cos\theta = -[a^{1/2}] / [a - a^{1/2} + 1]$$

where a is the mean distance of the planet from the Sun, in astronomical units.

Note: 1 astronomical unit = Earth's mean distance from the Sun.

Table 9.4 gives the *stationary* values of θ, according to different Siddhāntic texts as compared to the modern values.

Table 9.4 Stationary points for planets

Planet	**Mean distance a in ast.unit**	θ **Modern**	θ **Sūrya-siddhānta**	θ **Bhāskara & Lalla**	θ **Brahma-gupta**
Kuja	1.52369	163.215°	164°	163°	164°
Budha	0.3871	144.427°	144°	145°	146°
Guru	5.20256	125.565°	130°	125°	125° (*BS*) 130° (*KK*)
Śukra	0.7233	167.005°	163°	165°	165°
Śani	9.55475	114.466°	115°	113°	116°

Note: Brahmagupta has given the stationary value of θ for Guru as 125° in the **Brahmasphuṭasiddhānta** (BS) and as 130° in his **Khaṇḍa-khadyāka** (*KK*).

The stationary points θ, given in Table 9.4, are those at which the respective planets change their motion from *direct* to *retrograde*, i.e., the beginning of the retrograde motion (*vakrarambha*). The other stationary points, where the retrograde motion ends (*vakratyaga*), are given by (360° − θ).

It is noteworthy that Bhāskara II gives the correct value $\theta = 167°$ for *Śukra* in his **Karaṇa Kutūhalam**.

9.8 *BHUJĀNTARA* CORRECTION FOR STAR-PLANETS

As we have noted, in section 8.5, the true midnight at a place differs from the mean midnight by an amount of time called the "equation of time".

The correction to the celestial longitude of a planet, due to the role of the equation of time caused by the eccentricity of the Earth's orbit is called the *bhujāntara* correction.

As pointed out earlier, in section 8.5, in the case of the five star-planets also,

bhujāntara correction
= (eqn. of centre of the Sun)(daily motion of the planet) / 360°

where the equation of centre is in degrees.

If the daily motion of the planet is in *minutes* of arc, then,

bhujāntara correction (in minutes)
= (eqn. of centre in min.)(planet's daily motion in min.)/21600

Example: Find the *bhujāntara* correction for *Śani* at the midnight preceding March 22, 1991, at Bangalore.

We have, for the given date and time,

The Sun's equation of the centre = $+2°7'32'' = 127.53'$
Sani's true daily motion = $4'27'' = 4.45'$

Therefore,

$$\begin{aligned} \textit{bhujāntara}\text{ correction} &= (127.53')\,(4.45')\,/21600' \\ &= 0.0262735' \\ &= 1.576'' \end{aligned}$$

In fact, if the mean daily motion of *Śani*, viz., $2'$ is taken, then the *bhujāntara* correction would be $0.7085''$. Either way, this correction is negligible for most of the planets. However, in the case of the Moon the *bhujāntara* correction is quite pronounced.

10

Tripraśna—Direction, Place and Time

10.1 Introduction

In all Indian astronomical texts, the subject of finding directions, place and time from the shadow of the gnomon (*saṇkucchāyā*) occupies an important place. Usually, this chapter in these texts is called *Tripraśnādhikāra.*

10.2 Determination of the North–South line

Suppose O is the position of the gnomon (a vertical pole of a fixed height, *saṅku*). Draw a circle with O as the centre (see Fig.10.1).

Let W_1 be the point where the shadow of the gnomon enters into the circle in the forenoon, and E_1 be the point where the shadow passes out of the circle in the afternoon. This means W_1 and E_1 are the points

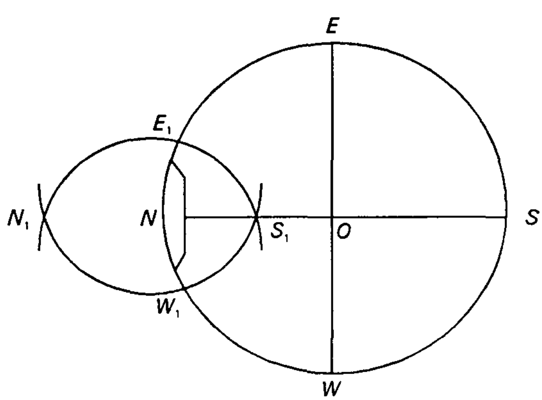

Figure 10.1 North–South line

where the end of the shadow of the gnomon (*saṅkucchāyāgra*) meets the circle in the forenoon and the afternoon, respectively. Join E_1W_1. The line E_1W_1 is directed from east to west.

With E_1 as the centre and with a radius greater than (½) E_1W_1 draw an arc of a circle; and with W_1 as centre and the same radius draw another arc cutting the former arc at the points N_1 and S_1. Join N_1 and S_1; let the line N_1S_1 cut the circle at N and S and the line through O, drawn parallel to E_1W_1, at E and W. Then E, W, N and S are the east, west, north and south directions with respect to O. However, since the declination of the Sun changes during the day, EW is not the exact east–west line always. Only on the two solstitial days, December 22 and June 22, EW represents the *exact* east–west line, because the change in declination on those days is negligible.

10.3 Finding the Latitude and Co-latitude of a Place

Usually, a gnomon is a vertical, cylindrical pole of a length of 12 *aṅgulas* (inches) with graduations at equal distances marked on it. However, the gnomon can be of any length, preferably quite long with many graduations for obtaining fairly accurate results.

The shadow of the gnomon at the noon of a place on the equinoctial day (i.e., March 22 or September 23) is referred to as the equinoctial midday shadow of the place.

Let OY be the gnomon erected at O (the local place), perpendicular to the plane of the horizon (Fig.10.2).

Let RD be perpendicular to the plane of the horizon and YX parallel to RO. From the two similar, right-angled triangles YOX and RDO, we have

$$DO / OX = RD / YO = RO / YX \qquad \ldots \qquad (1)$$

But,

$RO = R$, the radius of the celestial sphere,
$O\hat{Y}X = D\hat{R}O = \phi$, latitude of the place,
$OX = s$, length of the shadow
$OY = g$, length of the gnomon

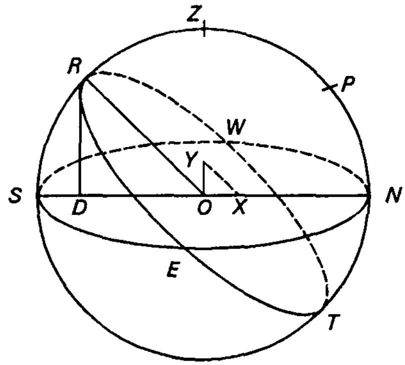

Figure 10.2 Latitude of a place

$$YX = \sqrt{g^2 + s^2} \text{ (from the triangle } OXY)$$
$$DO = R \sin \phi$$
$$RD = R \sin (90^\circ - \phi) = R \sin C$$

where $C = (90^\circ - \phi)$ is the "co-latitude" of the place. Substituting these in (1), we get

$$R \sin \phi \,/\, s = (R \sin \mathrm{C}) \,/\, g = R \,/\, \sqrt{g^2 + s^2}$$

or

$$R \sin C = g R \,/\, \sqrt{g^2 + s^2} \quad \ldots \quad (2)$$

and,

$$R \sin \phi = s R \,/\, \sqrt{g^2 + s^2} \quad \ldots \quad (3)$$

Dividing (3) by (2) and noting that $\sin C = \cos \phi$, we get

$$\tan \phi = s \,/\, g \quad \ldots \quad (4)$$

10.4 Rising and Setting Points of the Sun (Variation with Declination)

Since the declination δ of the Sun varies in the course of the year, the rising and setting points of the Sun also go on changing during the year.

The Sun remains in the southern hemisphere of the celestial sphere, with respect to the celestial equator, for half the year. Then, the Sun's declination is said to be southern and it is at its maximum around December 22. This is the time of the *winter solstice* for observers in the northern hemisphere of the Earth (and the *summer solstice* for those in the southern hemisphere). From that day onwards, the Sun takes a northern course (or *uttarāyaṇa*). It crosses the celestial equator and enters the northern hemisphere. Around March 22, its declination changes from south to north ($\delta = 0$ on the celestial equator). This point of crossing the celestial equator is called the *spring equinox* (for those in the southern hemisphere of the Earth, it is the *autumnal equinox*). The word "equinox" means equal day and night. At the spring and autumnal equinoxes (i.e., March 22 and September 23), the durations of the day and the night are equal throughout the world.

For the remaining half of the year, after March 22 the Sun's declination remains northern (positive) and attains the maximum around June 22, the *summer solstice* (for those in the southern hemisphere, it is the *winter solstice*). Then, the Sun starts its southward journey (*dakṣināyana*). The maximum declination of the Sun is about 23°27′, northern (+ve) on June 22 and southern (–ve) on December 22.

Connecting the azimuth A, altitude a, declination δ and the latitude ϕ of the place, we have the relation:

$$\cos A = (\sin \delta - \sin \phi \sin a) / \cos \phi \cos a \qquad \ldots \qquad (5)$$

When the Sun is rising or setting, its altitude (above the horizon) is zero, i.e., $a = 0$. Using this in equation (5), we get

$$[\cos A]_{\substack{\text{rising or}\\ \text{setting}}} = \sin \delta / \cos \phi \qquad \ldots \qquad (6)$$

or

azimuth at rising or setting,

$$A = \cos^{-1}[\sin \delta / \cos \phi]$$

Example: Calculate the points of sunrise and sunset on May 15th, September 15th, and January 15th of the year 1970 at Bangalore ($\phi = 13°$ N), given the declinations of the Sun for those days as 18°47′ N, 3°10′ N and 21° 11′ S, respectively.

(i) ON MAY 15TH

$\delta = +18°47', \phi = 13°$, so that

$$\cos A = \sin(+18°47') / \cos 13°$$
$$= 0.3219903 / 0.97437 = 0.33046$$
$$A = 70°\,42'\,11''$$

That is, the Sun rises at the point $70°42'11''$, from the north point eastward (or about $19°18'$ towards the north of the due east).

(ii) ON SEPTEMBER 15TH

$\delta = 3°10'\mathrm{N}, \phi = 13°$, so that

$$\cos A = \sin(+3°10') / \cos 13°$$
$$= 0.0552406 / 0.97437 = 0.566936$$

so that

$$A = 86°44'59''$$

i.e., the Sun rises at the point $86°45'$ eastward from the north and sets at the point $86°45'$ westward from the north (or rises and sets $3°15'$ northwards respectively from the east and the west points).

(iii) ON JANUARY 15TH

$\delta = -21°11', \phi = 13°$, so that

$$\cos A = \sin(-21°11') / \cos 13°$$
$$= -0.3613533 / 0.97437$$
$$= -0.3708584$$
$$= \cos(180° - 68°13'53'')$$

so that

$$A = 111°46'07''$$

i.e., the Sun rises at the point $111°46'$ eastward from the north point; this is, $21°46'$ away from the due east towards south.

Note: (i) For places on the equator, $\phi = 0$, in which case the azimuth of the rising or setting Sun is given by

$$\cos A = \sin\delta = \cos(90° - \delta)$$

or

$$A = 90^\circ - \delta$$

(ii) On March 22, $\delta = 0$, and hence $A = 90^\circ$; i.e., the sunrise takes place exactly at the east point. The same is the case on September 23.

10.5 Times of Sunrise and Sunset

If H is the hour angle, we have

$$\cos H = (\sin a - \sin\phi \sin\delta) / \cos\phi \cos\delta \qquad \dots \qquad (7)$$

At the times of sunrise and sunset, the Sun's altitude, $a = 0$, so that (7) becomes

$$\cos H = -\tan\phi \tan\delta \qquad \dots \qquad (8)$$

On a given day at a given place, the Sun's declination δ and the latitude of the place ϕ are known. From these, the hour angle H of the Sun at rising or setting can be determined.

The hour angle H is expressed in units of time (hours, minutes, seconds) at the rate of 15° per hour. This time *subtracted* from the local mean noon (12'o clock) gives the time of sunrise (local mean time). Similarly, the same value *added* to the local mean noon gives the local mean time of the local sunset.

The duration of the day $= 2H/15$ hrs and the duration of the night $= 24 - (2H/15)$ hours, where H is the hour angle (in degrees) obtained from (8).

Example: Find the times of sunrise and sunset and also the duration of day and night at Bangalore ($\phi = 13^\circ$N) on May 15, given that the Sun's declination is 18° 47′N.

We have

$$\cos H = -\tan(+13^\circ)\tan(+18^\circ 47') = -0.078519$$
$$\therefore H = 95.503444^\circ = 6\text{h } 18\text{m } 01\text{s}$$

Therefore, we have

(i) time of sunrise = 12h − 6h 18m 01s
= 5h 41m 59s a.m.

(ii) time of sunset = 12h + 6h 18m 01s
= 6h 18m 01s p.m.

(iii) duration of day, $2H$ = 2 (6h 18m 01s)
= 12h 36m 02s

(iv) duration of night = 24h – 12h 36m 02s
= 11h 23m 58s

Note: As pointed out earlier, for places in the northern hemisphere of the Earth, the duration of day is the longest, and the night shortest, on June 22. On that day, the declination of the Sun is maximum ($23°\,27'$ N). For places in the southern hemisphere, it is the other way round, i.e., on June 22, the duration of the day is shortest and that of the night is longest. On December 22 ($\delta = 23°$ S $27'$), the situation is reversed.

Example: Find the durations of the longest day and the shortest night at London (latitude: $51°\,32'$ N).

We have the hour angle of the Sun, H, at the time of sunrise or sunset given by

$$\cos H = -\tan\phi \tan\delta$$

Here, $\phi = +51°32'$ and $\delta = +23°27'$, so that

$$\begin{aligned}\cos H &= -\tan(51°32')\tan(23°27')\\ &= -(1.2586747)(0.4337751)\\ &= -0.5459817\end{aligned}$$

Therefore,

$$\begin{aligned}H &= 180° - 56.908222° = 123.09178°\\ &= 12\text{h} - (3\text{h } 47\text{m } 37.9\text{s})\\ &= 8\text{h } 12\text{m } 22.1\text{s}\end{aligned}$$

Therefore, at London, on June 22,

(i) the local time of sunrise = 12h – (8h 12m 22)
= 3h 47m 38s a.m.

(ii) the local time of sunset = 12h + (8h 12m 22s)
= 8h 12m 22s p.m.

(iii) the duration of day, $2H$ = 16h 24m 44s

(iv) the duration of night = 24h – $2H$
= 7h 35m 16s

10.6 THE RISING OF SIGNS OF THE ZODIAC

The point of the ecliptic rising at the eastern horizon at a given time at a place is called the *lagna* (or ascendant or the ecliptic orient point).

In Fig. 10.3, ♈ is the first point of Aries being one of the two points of intersection of the ecliptic with the celestial equator. Let X be a celestial body (or a point). The meridian through X cuts the celestial equator at D. Then, by definition, the arc ♈D (= $♈\hat{P}X$) is the right ascension (R.A.) of X, measured eastwards from ♈, from 0h to 24h in the time units. The direction of measurement of the right ascension is opposite to that of the hour angle of X. From Fig. 10.3, we have

$$\text{arc } R♈ = \text{arc } RD + \text{arc } ♈D$$

where R is the intersection of the meridian of the observer with the celestial equator. Arc RD corresponds to $Z\hat{P}X$ which is the hour angle of X and arc $R♈$ corresponds to $Z\hat{P}♈$ which is the hour angle of ♈.

The hour angle of ♈, the first point of Aries, is called the *Sidereal Time* (S.T.), i.e.,

$$Z\hat{P}♈ = P♈ = \text{S.T. (measured westward)}$$

Thus, we have

$$\text{Sidereal time} = \text{Hour angle of } ♈ = \text{Hour angle of } X + \text{R.A. of } X$$

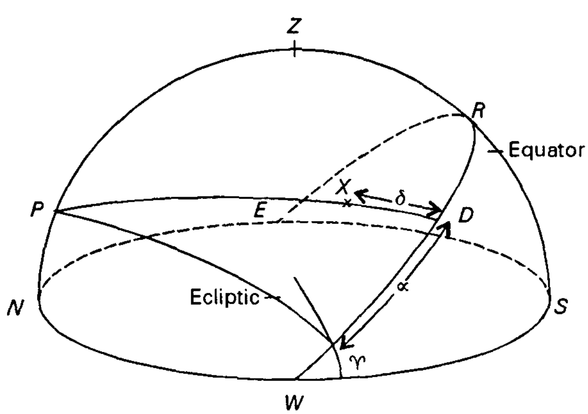

Figure 10.3 Right ascension and declination

or

$$\text{S.T.} = H + \alpha$$

If the S.T. at any instant is known, the R.A. (i.e., α) of the celestial object X can be determined from its hour angle H and vice versa. Now, let us take X as the point of the ecliptic intersecting the eastern (celestial) horizon. Then, at that instant, we have

$$\begin{aligned}\text{S.T.} &= \text{Hour angle of } \Upsilon \\ &= \text{Hour angle of } X + \text{H.A. of } X\end{aligned}$$

But,

$$\text{Hour Angle of } X = -Z\hat{P}X = -H_t$$

The negative sign is taken on the right side since X is to the east of Z.

For the point on the ecliptic (latitude $\beta = 0$) at the time of its rising (altitude $a = 0$), we have:

$$\cos(H_t) = -\tan\phi \tan\delta \qquad \ldots \qquad (1)$$

and

$$\sin\alpha = \tan\delta \cot\varepsilon \qquad \ldots \qquad (2)$$

where ϕ is the latitude of the place and ε is the obliquity of the ecliptic (about 23° 27′). From (1) and (2), we get:

$$\text{S.T.} = -\cos^{-1}(-\tan\phi \tan\delta) + \sin^{-1}(\tan\delta / \tan\varepsilon)$$

Therefore, the S.T. at the instant of the rising of the ecliptic point, having declination δ, can thus be calculated.

In practice, one can determine the difference in sidereal times corresponding to the rising of two points of the ecliptic at the eastern horizon at a given place (latitude ϕ). This gives us the duration of each interval of the ecliptic (like *rāśi*) rising in the eastern horizon.

Example: Find the rising time of the first points of *nirayana Meṣa, Vṛṣabha, Mithuna,* etc., on June 22, 1991, at Madras (latitude 13° N).

The *ayanāṁśa* as on June 22, 1991 is 23°44′33″ (according to the *Indian Ephemeris*). Let λ_1, λ_2, etc., be the (tropical or *sāyana*) longitudes of the first points of *Meṣa, Vṛṣabha,* etc., and δ_1, δ_2, etc., be their declinations.

Then, we have:

$$\sin\delta_1 = \sin\lambda_1 \sin\varepsilon \qquad \ldots \qquad (3)$$

for i = 1, 2,...12 and $\varepsilon = 23^\circ 27'$. Now, we have, adding 30″ successively,

$$\lambda_1 = 23^\circ 44' 32'',\ \lambda_2 = 53^\circ 44' 32'',\ \lambda_3 = 83^\circ 44' 32'' \text{ etc.}$$

Using these values of λ_i, the corresponding values of δ_i can be determined.

$$\delta_1 = 9^\circ.21984,\quad \delta_2 = 18^\circ.71625,\quad \delta_3 = 23^\circ.30199$$

Then, the hour angles of the first points are

$$H_i = -\cos^{-1}(-\tan\phi \tan\delta_i) \text{ in degrees}$$
$$|H_i| = \cos^{-1}(-\tan\phi \tan\delta_i)$$

and the right ascensions are

$$\propto_i = \sin^{-1}(\tan\delta_i \cot\varepsilon)$$

so that the sidereal times of the rising of the first points of *nirayana Meṣa*, etc., are given by

$$S_i = H_i + \propto_i$$

in angles which can be converted into time units by dividing them by 15. Then, $(S_i - S_1)$ is the sidereal time interval between the rising of the first point of *Meṣa* and that of the i^{th} *rāśi.* This S.T. interval is converted into a civil time interval by dividing it by 1.002738, in which case we get $T_i - T_1$. When this time interval is added to the time of the rising of the first point of *Meṣa*, we obtain the rising time of the i^{th} *rāśi.* Consider

$$\lambda_{i+6} = 180^\circ + \lambda_i$$

Therefore,

$$\sin(\lambda_{i+6}) = -\sin\lambda_i$$

Also,

$$\sin(\delta_{i+6}) = \sin(\delta_{i\pm6}) \sin\varepsilon$$
$$= -\sin\lambda_i \sin\varepsilon = -\sin\delta_1$$

i.e.,

$$\delta_{i+6} = -\delta_i$$

for instance, substituting λ_i's in Eqn. (3), we get

$$\begin{aligned}\delta_7 &= -9.21934^\circ = -\delta_1\\ \delta_9 &= -23.30193^\circ = -\delta_3\end{aligned}$$

Similarly, consider

$$\begin{aligned}|H_{i+6}| &= \cos^{-1}(-\tan\phi\tan\delta_{i+6})\\ &= \cos^{-1}[-\tan\phi\tan(-\delta_i)]\\ &= \cos^{-1}(\tan\phi\tan\delta_i)\end{aligned}$$

Therefore,

$$\cos|H_{i+6}| = \tan\phi\tan\delta_i = -\cos|H_i|$$

or

$$|H_{i+6}| = 180^\circ - |H_i|$$

or

$$|H_{i+6}| = 180^\circ + |H_i|$$

Due to the geometry of the hour angle at rising, the magnitude of the hour angle cannot be greater than 180°. Therefore,

$$|H_{i+6}| = 180^\circ - |H_i| = 180^\circ + |H_i|$$

or

$$H_{i+6} = -(180^\circ + H_i)$$

For example, we get

$$\begin{aligned}H_7 &= -87.85236\\ &= -(180^\circ - 92.14764^\circ)\\ &= -(180^\circ + H_1)\end{aligned}$$

i.e., $H_1 = -92^\circ.14764$

Also,

$$\begin{aligned}\propto_{i+6} &= \sin^{-1}(\tan\delta_{i+6}\cot\varepsilon)\\ &= \sin^{-1}(-\tan\delta_i\cot\varepsilon),\ \text{since } \delta_{i+6} = -\delta_i\end{aligned}$$

Therefore,

$$\sin \alpha_{i+6} = -\tan \delta_i \cot \varepsilon = -\sin \alpha_i$$

This means that

$$\alpha_{i+6} = 180^\circ + \alpha_i$$

or

$$\alpha_{i+6} = 360^\circ - \alpha_i$$

Again, from the geometry, it follows that

$$\alpha_{i+6} = 180^\circ + \alpha_i$$

For example,

$$\begin{aligned} \alpha_7 &= \text{R.A. of } \textit{Tulā} \text{ first point} \\ &= 180^\circ + \text{R.A. of } \textit{Meṣa} \text{ first point} \\ &= 180^\circ + 21.97377^\circ \\ &= 201.97377^\circ. \end{aligned}$$

10.7 Intervals of Rising of *Sāyana Rāśis* (or Signs)

According to the **Khaṇḍakhādyaka**, the durations of the rising of the *sāyana Meṣa, Vṛṣabha* and *Mithuna* at Laṅkā (on the equator) are, respectively, 278,299 and 323 *vināḍīs*. These, *diminished* by the *vināḍīs* of the local ascensional differences, are the durations of the risings of these three *rāśis* at one's own place.

The figures written in the reverse order *increased* by the ascensional differences (*cara*), are the durations of the risings of the next three signs at the observer's place. We have

$$\sin (\textit{cara}) = \tan \phi \tan \delta$$

where ϕ is the latitude of the place and δ the declination corresponding to the ending of the *rāśi*, viz., for longitudes $\lambda_1 = 30^\circ, \lambda_2 = 60^\circ$ and $\lambda_3 = 90^\circ$. Also, with latitude $\beta = 0$ (for the points on the ecliptic), we have:

$$\sin \delta = \sin \lambda \sin \varepsilon \ (\varepsilon = 24^\circ)$$

Now, for $\lambda_1 = 30°, \lambda_2 = 60°$ and $\lambda_3 = 90°$, with the obliquity of the ecliptic ε being taken as 24°, we get the corresponding values of the declination:

$$\begin{aligned}
\delta_1 &= \sin^{-1}[\sin 30° \sin 24°] = 11.734° \\
\delta_2 &= \sin^{-1}[\sin 60° \sin 24°] = 20.624646° \\
\delta_3 &= \sin^{-1}[\sin 90° \sin 24°] = 24°
\end{aligned}$$

The tabular differences of *cara* (ascensional difference) for a place of latitude ϕ, are given by the successive differences of

$R \tan \phi \tan \delta_i$ (*asus*)

where

1 *vināḍīka* = 6 *asus*

Therefore, corresponding to the declinations δ_i of the ending of the first 3 *rāśis*, the tabular differences of *cara*, are given by the successive differences of

$(R \tan \phi \tan \delta_i) / 6$ (*vināḍīkas*)

Thus, for example, for Chennai or Bangalore ($\phi = 13°$), we have the tabular differences given by the successive differences of

(i) $\tan 13° (\tan 11.734°) \times 3438 / 6$
$= 27.5$ *vin.*

(ii) $\tan 13° (\tan 20.625°) \times 3438 / 6$
$= 49.79$ *vin.*

(iii) $\tan 13° (\tan 24°) \times 3438 / 6$
$= 58.90$ *vin.*

Therefore, the tabular differences of the *cara* for the first three *rāśis* at Chennai (or Bangalore) are respectively, 27.5, (49.79 – 27.5), (58.9 – 49.79), i.e.,

27.5, 22.29 and 9.11 *vināḍīs.*

The durations of the risings of the twelve *rāśis* at Chennai calculated according to the **Khaṇḍakhādyaka**, are given in Table 10.1.

Table 10.1 Durations of risings of *rāśis* at Chennai or Bangalore ($\phi = 13°$)

Rāśi	**Durations of risings at Laṅkā (*vināḍīs*)**	**Ascensional difference (tabular diff.)**	**Durations of risings at Chennai or Bangalore (*vināḍīs*)**
Meṣa	278	–27.5	250.5
Vṛṣabha	299	–22.29	276.71
Mithuna	323	–9.11	313.89
Karkaṭaka	323	+9.11	332.11
Simha	299	+22.29	321.29
Kanyā	278	+27.5	305.5
Tulā	278	+27.5	305.5
Vṛścika	299	+22.29	321.29
Dhanus	323	+9.11	332.11
Makara	323	–9.11	313.89
Kumbha	299	–22.29	276.71
Mīna	278	–27.5	250.5

Note: (1) The tabular differences are *additive* for the *rāśis* from *Karkaṭaka* ($\lambda = 90°$) to the end of *Dhanus* ($\lambda = 270°$), and *negative* otherwise. (2) The total of the durations of risings of all the twelve *rāśis* is 60 *nāḍīs*.

10.8 Determination of *Lagna* at a Given Time and Place

The Sun's longitude increases proportionately from the time in *ghaṭikās* (or *nāḍikās*) elapsed since sunrise, at the given place on the given day, by means of local time durations for the risings of the *rāśis*, and becomes the *lagna* (or orient ecliptic point or the ascendant). Again, conversely, by making the longitude of the Sun equal to the orient ecliptic point by the local time intervals for the risings of the *rāśis*, the time elapsed since sunrise can be obtained.

Note: From the *sāyana lagna*, thus obtained, subtract the *ayanāmśa* to determine the *nirayana lagna*.

Examples: Find the *lagna* at 5 *ghaṭikās* elapsed since sunrise at Bangalore, given the *sāyana* longitude of the Sun as 11s 19°46′36″ at that instant.

The Sun is in *sāyana Mīna rāśi* with the remainder of

$$= 30° - (19°46'36'')$$
$$= 10°13'24'' = 613.4'$$

The duration of the rising of *sāyana Mīna* is 250.5′ *vināḍīs* (Table 10.1). Therefore, the time taken for the rising of 613.4′ is

$$(613.4')(250.5) / (30 \times 60) = 85.365 \textit{ vināḍīs}.$$

The given time elapsed since sunrise $= 5$ *gh.*
$= 300$ *vināḍīs*

Now, out of 300 *vināḍīs* substracting 85.365 *vināḍīs* for the residue of *Mīna*, we get

$$(300 - 85.365) = 214.635 \textit{ vin.}$$

The duration of the rising of the next *rāśi* (*Meṣa*) is 250.5 *vin.* (see Table 10.1). Therefore, 214.635 *vin.* corresponds to

$$(214.635 / 250.5) \times 30° = 25°42'17''$$

of *Meṣa*. Therefore, we have

Lagna = *sāyana Meṣa* 25° 42′ 17″
= 0s 25° 42′ 17″

one can get more accurate values by making use of equations (1), (2) and (3).

11

Lunar Eclipse

11.1 Cause of Lunar Eclipses

On a full moon day, the Sun and the Moon are on the opposite sides of the Earth. The Sun's rays fall on the side of the Earth facing the Sun, and a shadow will be cast on the other side. When the Moon enters the shadow of the Earth, a lunar eclipse occurs. This happens when the Sun and the Moon are in *opposition,* i.e., the difference between the longitudes of the Sun and the Moon is 180°.

However, a lunar eclipse does not occur on every full moon day. This is because the plane of the Moon's orbit is inclined at about 5° to the ecliptic. If the Moon's orbit were in the plane of the ecliptic, then there would have been a lunar eclipse every full moon day. Generally, on a full moon day, the Moon will be either far above or far below the plane of the ecliptic and so does not pass through the shadow of the Earth. But, on that full moon day, when the Moon does pass through the Earth's shadow, a lunar eclipse occurs.

In order for a lunar eclipse to occur, the Moon must come close to the ecliptic. This means that the Moon, on the full moon day, must be close to one of the *nodes* of the Moon. In Fig.11.1, the orbit of the Moon intersects with the ecliptic at two points N and N'. These two points are referred to as the ascending and the descending nodes of the Moon. They are called *Rāhu* and *Ketu* in Indian astronomy.

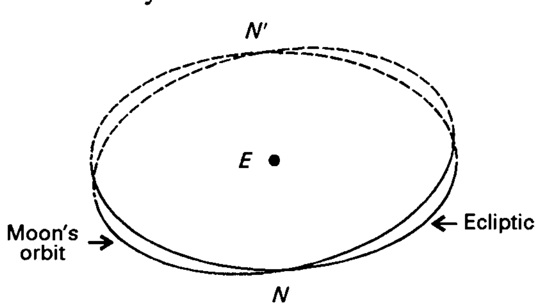

Figure 11.1 Nodes of the Moon

The lunar eclipse is said to be *total* when the whole of the Moon passes through the shadow. The eclipse is *partial* when only a part of the Moon enters the shadow.

In Fig. 11.2, *S* and *E* represent the centres of the Sun and the Earth, respectively. Draw a pair of direct tangents *AB* and *CD* to the surface of the Sun and the Earth, meeting *SE* in *X*. If these lines are imagined to revolve round *SE* as axis, they will generate cones. There is, thus, a conical shadow *BVD*, with *V* as its vertex, across which no direct ray from the Sun can fall. This conical space is called the *umbra*.

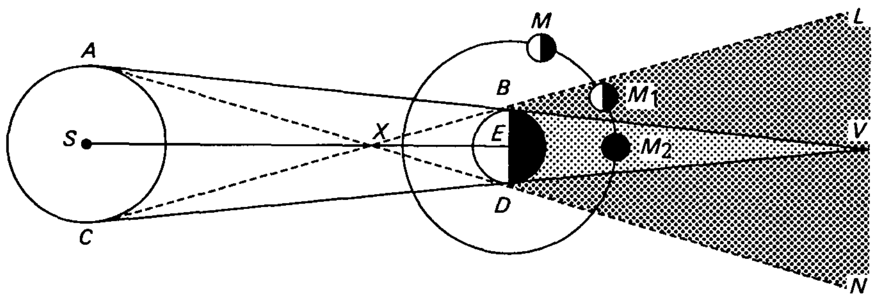

Figure 11.2 Earth's shadow-cone and the lunar eclipse

The spaces around the umbra, represented by *VBL* and *VDN*, form what is called the *penumbra*, from which only a part of the Sun's light is excluded. It is to be noted that the passage of the Moon through the penumbra does not prompt an eclipse. It results only in diminution of the Moon's brightness.

When the Moon is at M_1 (see Fig. 11.2) it receives light from portions of the Sun next to *A*, but rays from the parts near *C* will not reach the Moon at M_1. Therefore, the brightness is diminished, the diminution growing greater as the Moon approaches the edge of the umbra. An eclipse is considered as just commencing when the Moon enters the umbra or the shadow-cone.

11.2 Angular Diameter of the Shadow-cone

In Fig. 11.3, the angular diameter of the cross-section of the shadow-cone is represented by an arc *MN*. Let the

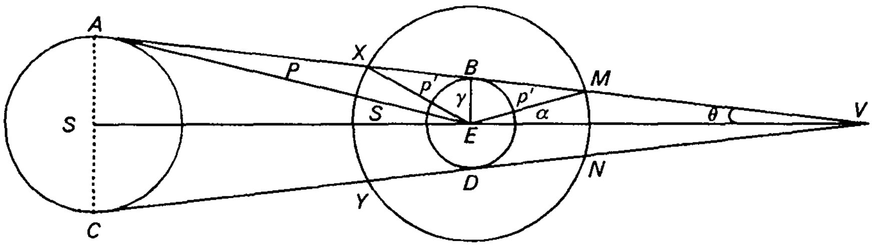

Figure 10.1 North–South line

semi-angle *MEV*, subtended by *MN* at the centre of the Earth, be $\propto$. We have

$$
\begin{aligned}
p &= \text{Sun's horizontal parallax} = E\hat{A}X \\
p' &= \text{Moon's horizontal parallax} = E\hat{M}B = E\hat{X}B \\
s &= \text{Sun's angular semi-diameter} = S\hat{E}A \\
\theta &= \text{semi-vertical angle of the shadow-cone} = E\hat{V}B
\end{aligned}
$$

Now, in triangle *MEV*, we have : $\propto + \theta = p'$ so that

$$\propto = p' - \theta \qquad \ldots \qquad (1)$$

Similarly, we have, from triangle *AEV*,

$$\theta = s - p \qquad \ldots \qquad (2)$$

From (1) and (2) we get

$$\propto = p' - (s - p) \qquad \text{or} \qquad \propto = p + p' - s \qquad \ldots \qquad (3)$$

Since p, p' and s are known, the angular semi-diameter $\propto$ of the shadow-cone is determined using (3). However, it is found that the Earth's atmosphere increases, due to absorption, the calculated radius of the shadow-cone by about two percent. Therefore, for the prediction of lunar eclipses, the following expression is used:

$$\propto = (51/50)(p + p' - s) \qquad \ldots \qquad (4)$$

As an example, using the mean values, we have:

Moon's hor. parallax, $p' = 57'$
Sun's hor. parallax, $p = 8''$ and
Sun's semi-diameter, $s = 16'$

the angular semi-diameter of the shadow-cone, from (4), is given by

$$\alpha = (51/50)\,[8'' + 57' - 16']$$

or

$$\alpha = 41'.956 \approx 42'$$

To be more accurate, it is found that α varies from a minimum of $37'49''$ to a maximum of $44'37''$. In fact, the maximum value of α is reached when the Moon is nearest to the Earth (i.e., perigee) and the Earth itself being at the same time farthest from the Sun (aphelion or apogee of the Sun), i.e., when p' is maximum and s is the minimum value attained when the conditions are reversed, i.e., when the Moon is farthest from the Earth (i.e., at apogee) and the Earth itself, at the same time, being closest to the Sun i.e., when p is minimum and s maximum.

11.3 Ecliptic Limits for the Lunar Eclipse

We have noted earlier that the possibility of an eclipse on a full moon day is restricted, due to the inclination of the Moon's orbit to the plane of the ecliptic.

In Fig. 11.4, NM represents the Moon's orbit; M and C are the centres of the Moon and the shadow, respectively, when the eclipse is

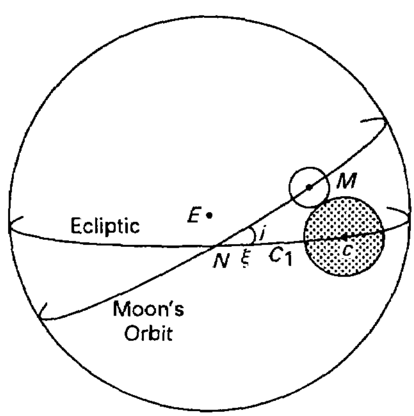

Figure 11.4 Ecliptic limits

about to take place. Let C_1 be the position of the shadow's centre when the Moon is at the node *N*. Let NC_1 be denoted by ξ and *t* be the time required by the Moon to go from *N* to *M* and for the shadow's centre to go from C_1 to *C*.

The geocentric longitude of *C* is the Sun's longitude plus 180°. Hence, in finding the maximum value of ξ in which an eclipse is possible, we are actually finding the maximum angular distance of the Sun from the other node.

Suppose the Sun's longitude increases at the rate of θ, and ϕ is the angular velocity of the Moon in its orbit. For simplicity, let us take θ and ϕ constants. Then, we have:

$$NM = \phi\, t \text{ and } NC = \xi + \theta\, t$$

If η is the angular distance *CM* and *i* the inclination $M\hat{N}C$, we have from triangle *MNC* (regarded as a plane triangle),

$$CM^2 = NC^2 + NM^2 - 2NC.\, NM \cos i$$

$$\text{i.e.,}\quad \eta^2 = (\xi + \theta t)^2 + (\phi t)^2 - 2(\phi t)\,(\xi + \theta t) \cos i$$

or

$$\eta^2 = \xi^2 - 2\xi t\,(\phi \cos i - \theta) + t^2\,[\theta^2 + \phi^2 - 2\theta\phi \cos i]$$

Now, η is *minimum* when *t* is given by:

$$\xi\,(\phi \cos i - \theta) - t\,[\theta^2 + \phi^2 - 2\theta\phi \cos i] = 0$$

In fact, we have:

$$\eta_{min} = \eta_o = \xi\, \phi \sin i \,/\, [\theta^2 + \phi^2 - 2\,\theta\,\phi \cos i]^{1/2} \qquad \ldots \qquad (1)$$

Let $q = \theta \,/\, \phi$, so that (1) becomes

$$\xi = \eta_o\,[\,1-2\,q \cos i + q^2]^{1/2} \operatorname{cosec} i \qquad \ldots \qquad (2)$$

In (2), *q* is the ratio of the Earth's orbital angular velocity to that of the Moon, which is the same as the ratio of the Moon's sidereal period to the duration of a year. Taking the mean values, we have

$$q \approx 3 \,/\, 40$$

Also, since $i \approx 5.2°$, from (2), we get

$$\xi = 10.3\, \eta_o \qquad \ldots \qquad (3)$$

When the Moon is about to enter the umbra shadow-cone, we have

$$\eta_o = \propto + s'$$

where $\propto$ is the angular semi-diameter of the shadow-cone and s' is the Moon's angular semi-diameter. We have from (4) of the previous section,

$$\propto = (51/50)(p + p' - s)$$

For a *partial* lunar eclipse to be possible, it is evident that

$$\xi \leq 10.3(\propto + s')$$

For a *total* lunar eclipse,

$$\xi \leq 10.3(\propto + s')$$

For example, taking the following values:

s = Sun's semi-diameter = $16' = 960''$
s' = Moon's semi-diameter = $15'35'' = 935''$
p = hor. parallax of the Sun = $9''$
p' = hor. parallax of the Moon = $3422''$

(i) to satisfy the condition for a partial eclipse of the Moon, we must have

$$\xi < 10.3\,[(51/50)(p + p' - s) + s']$$
$$= 10.3\,[(51/50)(9'' + 3422'' - 960'') + 935''] = 9.8863°$$

or

$$\xi < 9.9°$$

(ii) the conditions for a total lunar eclipse:

$$\xi < 10.3\,[(51/50)(p + p' - s) - s]$$

or

$$\xi < 10.3\,[(51/50)(9'' + 3422'' - 960'') - 935'']$$
$$= 10.3\,[(51/50)(2471'') - 935''] = 4.5361°$$

or

$$\xi < 4.6°$$

These values of ξ are called the *ecliptic limits* for the occurrence of a lunar eclipse.

However, since the quantities used in the above derivation are only mean, considering the actual variations, it is found that for a *partial eclipse* the *maximum* value of ξ is 12.1° and the *minimum* value of ξ is 9.5°, which are respectively called the *superior* and *inferior* ecliptic limits.

11.4 Half-durations of Eclipse and of Maximum Obscuration

The next important step is to determine the instants of the beginning and the end of a lunar eclipse as also of the maximum obscuration. For this, we need to find the duration of the first half and the second half of the total duration of the eclipse. This is explained in Fig.11.5.

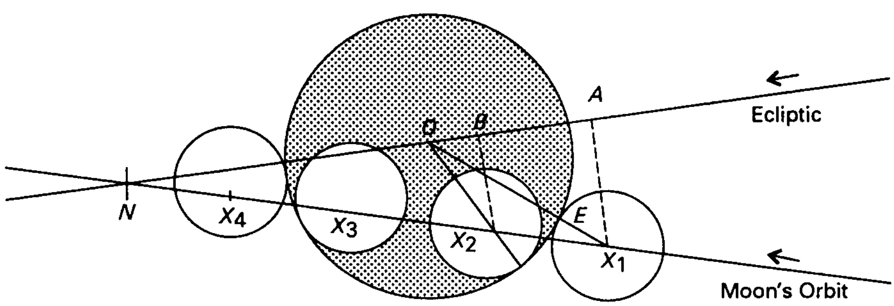

Figure 11.5 Half-duration of lunar eclipse

A half-duration is the time taken by the Moon, relative to the Sun, so that the point *A* in the figure moves through *OA*. We have:

$$\begin{aligned} OA_2 &= OX^2_1 - AX^2_1 = (OE + EX_1)^2 - AX_1^2 \\ &= (d_1 + d_2)^2 - \beta_1^2 \end{aligned}$$

where

$OE = d_1$ = Semi-diameter of the shadow
$EX_1 = d_2$ = Semi-diameter of the Moon
$\beta_1 = AX_1$ = Latitude of the Moon (*Vikṣepa*)

When the Moon's centre is at X_1, we have

$$\text{Half-duration} = \frac{\sqrt{(d_1 + d_2)^2 - \beta_1^2}}{(\text{Moon's daily motion} - \text{Sun's daily motion})}$$

Since the actual moment of the beginning of the eclipse, and hence the Moon's latitude then, are not known, the above formula is used iteratively.

By a similar analysis, the half-duration of maximum obscuration (or totality as the case may be) is given by

$$\text{Half-duration of max. obsn.} = \frac{\sqrt{(d_1 \sim d_2)^2 - \beta_1^2}}{(DM - DS)}$$

where *DM* and *DS* are the daily motions of the Moon and the Sun, respectively.

The, thus obtained, half-duration of the eclipse and the maximum obscuration are:

(1) subtracted from the instant of the opposition to get the first moments; and
(2) added to the instant of the opposition to obtain the last moments.

Finally, the magnitude (*pramāṇam*) of the eclipse is given by

$$\text{Magnitude} = \frac{\text{Amount of obscuration } (\textit{Grāsa})}{\text{Angular diameter of the Moon}}$$

Obviously, if the magnitude is greater than or equal to 1, then the eclipse is total; otherwise, it is partial.

It is also clear, from Fig.11.5, that if the sum of the angular semi-diameters of the Moon and the shadow is less than the latitude of the Moon, there will be no eclipse.

11.5 Lunar Eclipse According to Sūryasiddhānta

The procedure of the computation of a lunar eclipse is described in the **Sūryasiddhānta** (*SS*) in Chapter 4 (*Candra-grahaṇam*) of the text.

The parameters required for the computation of a lunar eclipse are:

(i) True longitudes of the Sun, the Moon and the Moon's node (*Rāhu*),
(ii) The true daily motions of these three bodies,
(iii) The latitude of the Moon and,
(iv) The angular diameters of the Earth's shadow (*Bhūcchāyā*) and of the Moon.

Example: The lunar eclipse on September 27,1996.

Since the longitude of the Moon's node (*Rāhu*) according to *SS* is not very accurate, we use the true longitudes of the Sun, Moon, etc., from the *Ind. Ast. Eph.* as at 5-30 a.m.(IST). However, the procedure of the *SS* is adopted.

True longitude of the Sun	=	$160° 21' 01''$
True longitude of the Moon	=	$338° 44' 27''$
True longitude of *Rāhu*	=	$164° 10' 14''$
True daily motion of the Sun	=	$58' 51''$
True half-daily motion of the Moon	=	$432' 32''.7$
True daily motion of the Moon	=	$861'$
Node's daily motion	=	$3' 11''$
Instant of opposition	=	8h 24m (IST)

(i) TO FIND THE SUN'S DIAMETER

$$\text{Sun's corrected diameter} = \frac{58' 51'' \times 6500^y}{58' 58''}$$
$$= 6487^y.13$$

$$\frac{43{,}20{,}000 \times 6487^y.13}{57{,}753{,}336} = 485^y.24$$

and $$\frac{485^y.24}{15} = 32' 20''$$

where $58'58''$ is the Sun's mean daily motion and 57,753,336 is the number of the Moon's revolutions in a *Mahāyuga* and 6500 *yojanas* is the Sun's mean diameter. Here y stand for *yojana*, a distance unit, with $1y = 15'$.

(ii) TO FIND THE MOON'S DIAMETER

$$\text{Moon's corrected diameter} = \frac{861' \times 480^y}{788'\ 25''} = 524^y.2$$

and $$\frac{524^y.2}{15} = 34'\ 56''$$

where 480^y is the Moon's mean diameter and $788'\ 25''$ is the Moon's mean daily motion.

(iii) TRUE LONGITUDES OF SUN, MOON AND NODE AT THE OPPOSITION

Interval from 5-30 a.m. to the instant of opposition : $2^h\ 54^m$

$$\text{Motion of the Sun in } 2^h\ 54^m = \frac{58'51'' \times 2^h54^m}{24} \cong 7'77''$$

$$\therefore \text{Sun's true longitude at opposition} = 160°21'01'' + 7'7''$$
$$= 160°28'08''$$

$$\text{Motion of the Moon in } 2^h\ 54^m = 2^h\ 54^m \times 432'\ 32''.7\ /\ 12^h$$
$$= 104'32'' = 1°\ 44'\ 32''$$

$\therefore$ True longitude of the Moon at opposition
$$= 338°\ 44'\ 27'' + 1°\ 44'\ 32'' = 340°\ 28'\ 59''$$

Node's longitude at opposition:

$$\text{Node's motion in } 2^h\ 54^m = \frac{3'11'' \times 2^h\ 54^m}{24^h} = 23''$$

$$\therefore \text{Node's true longitude} = 164°\ 10'\ 14'' - 23''$$
$$= 164°\ 9'\ 51''$$

(iv) DIAMETER OF THE EARTH'S SHADOW

Earth's corrected diameter

$$= \frac{\text{True daily motion of the Moon} \times 1600^y}{\text{Mean daily motion of the Moon}}$$

$$= \frac{861' \times 1600^y}{788'\ 25''} = 1747^y.2$$

Sun's corrected diameter – Earth's diameter

$$= 6487^y.13 - 1600^y = 4887^y.13$$

and $$\frac{480^y \times 4887^y.13}{6500^y} = 360^y.9$$

Diameter of the Earth's shadow

$= $ Earth's corrected diameter $- 360^y.9$

$= 1747^y.2 - 360^y.9$

$= 1386^y.3$

$\therefore$ Earth's shadow diameter (arc) $= 1386.3 / 15$

$= 92'25''$

(v) THE MOON'S LATITUDE AT THE MIDDLE OF THE ECLIPSE AND THE AMOUNT OF GREATEST OBSCURATION:

(Longitude of the Moon – Longitude of Node) at the opposition

$= 340^\circ\, 28'\, 59'' - 164^\circ\, 09'\, 51''$

$= 176^\circ\, 19'\, 08''$

Bhuja of the above difference $\cong 3^\circ\, 41'$

$\therefore$ *Jyā* $(3^\circ\, 41') = 221'$

The Moon's latitude at the instant of opposition

$$= \frac{270' \times 221'}{3438'}$$

$= 17'21''$

Here, it is assumed that the inclination of the Moon's orbit is $4^\circ\, 30'$. As determined earlier,

Semi-diameter of the eclipsed body (Moon) $= 17'28''$

Semi-diameter of the eclipsing body

(Earth's shadow) $= 46'\, 12''$

Their sum $= 63'\, 40''$

Deduct the Moon's latitude $= 17'\, 21''$

$\therefore$ Obscured portion (*grāsa*) $= 46'\, 19''$

Since *grāsa* is greater than the Moon's diameter, the lunar *eclipse* is *total*.

(vi) THE DURATION OF THE ECLIPSE AND OF TOTAL OBSCURATION

Diameter of the eclipsing body (shadow) :	$92' 25''$	$92' 25''$
Diameter of the eclipsed body (Moon) :	$34' 56''$	$34' 56''$
Their sum and difference :	$127' 21''$	$57' 29''$
Half-sum and half-difference :	$63' 40''$	$28' 44''$
Squares of the above :	4053	825
Deduct the square of the Moon's latitude :	301	301
	3752	524
Their square roots :	$61' 15''$	$22' 53''$

(a) Half-duration of the eclipse

$$= \frac{61' 15'' \times 60^n}{\text{(Daily motion of the Moon} - \text{Daily motion of the Sun)}}$$

$$= \frac{61' 15'' \times 60^n}{802' 9''} = 4^n 34^v$$

Note: The superscripts *n* and *v* denote *nāḍīs* and *vināḍīs*

(b) Half-duration of totality

$$= \frac{22' 53'' \times 60^n}{802' 9''} = 1^n 42^v$$

To get a more accurate value of the Moon's latitude more iterations have to be carried out

$$\text{Moon's motion in } 4^n 34^v = \frac{4^n 34^v \times 861'}{60^n} = 1° 5' 31''$$

$$\text{Node's motion in } 4^n 34^v = \frac{4^n 34^v \times 3' 11''}{60^n} = 14''$$

Moon's longitude at opposition :	$340° 28' 59''$	$340° 28' 59''$
Add and *subtract* the Moon's motion in $4^n 34^v$:	(+) $1° 05' 31''$	(–) $1° 05' 31''$
	$341° 34' 30''$	$339° 23' 28''$
Node's longitude at opposition :	$164° 09' 51''$	$164° 09' 51''$
Subtract and *add* the node's motion in $4^n 34^v$:	(–) $14''$	(+) $14''$
	$164° 9' 37''$	$164° 10' 05''$

Long. of Moon – Long. of Node :	177° 24′ 53″	175° 13′ 23″
Jyā values of the above :	158′	287′
Moon's latitude at the *end* and the *beginning* of the eclipse :	12′ 24″	22′ 32″
Squares of half the sum of the diameters :	4053	4053
Deduct the squares of latitudes :	153	507
	3900	3546
Their square roots :	62′ 26″	59′ 32″

$$\text{Corrected second half-duration} = \frac{60^n \times 62'\,26''}{802'\,9''} = 4^n\,40^v$$

$$\text{Corrected first half-duration} = \frac{60^n \times 59'\,32''}{802'\,9''} = 4^n\,27^v$$

(vii) APPARENT INSTANTS OF BEGINNING AND END OF TOTALITY

$$\text{Moon's motion in } 1^n\,42^v = \frac{1^n\,42^v \times 861'}{60^n} = 24'\,23''$$

$$\text{Node's motion in } 1^n\,42^v = \frac{1^n 42^v \times 3'\,10''}{60^n} = 5''$$

Moon's longitude at opposition :	340° 28′ 59″	340° 28′ 59″
Add and *subtract* the Moon's motion in $1^n 42^v$:	(+) 0° 24′ 23″	(–) 0° 24′ 23″
Moon's long. at the *end* and *beginning* of totality :	340° 53′ 22″	340° 04′ 36″
Node's longitude at opposition :	164° 09′ 51″	164° 09′ 51″
Subtract and *add* the node's motion in $1^n 42^v$:	(–) 5″	(+) 5″
Node's long. at the *end* and *beginning* of totality :	164° 09′ 46″	164° 09′ 56″
Moon's distance from node :	176° 42′	175° 53′
Jyā values :	197′	246′
Moon's latitude at the *end* and the *beginning* of the eclipse :	15′ 28″	19′ 19″

Squares of half-difference of the diameters of the Moon and shadow	:	825	825
Deduct the squares of latitudes	:	239	373
		586	454
Their square roots	:	24′.21	21′.26

$$\text{Second half-interval of totality} = \frac{24'.21 \times 60^{n}}{802'\ 9''}$$
$$= 1^{n}\ 49^{v}$$
$$= 0^{h}\ 43^{m}$$

$$\text{First half-interval of totality} = \frac{21'.26 \times 60^{n}}{802'\ 9''}$$
$$= 1^{n}\ 35^{v}$$
$$= 0^{h}\ 38^{m}$$

SUMMARY OF THE LUNAR ECLIPSE

Beginning of eclipse	:	$6^{h}\ 42^{m}$
Beginning of totality	:	$7^{h}\ 49^{m}$
Middle of eclipse	:	$8^{h}\ 24^{m}$
End of totality	:	$8^{h}\ 59^{m}$
End of eclipse	:	$10^{h}\ 06^{m}$

Note: The timings coincide exactly with the ones given in the *Ind. Ast. Eph.*

12

Solar Eclipse

12.1 How a Solar Eclipse is Caused

On a new moon day, the Sun and the Moon are on the same side of the Earth (see Fig.12.1). The rays of the Sun *S* which fall on the surface of the Moon *M*, facing the Sun, are prevented from reaching the Earth. A shadow-cone is caused by the Moon on the side facing the Earth. A solar eclipse is caused under the following conditions:

(i) the Sun and the Moon must be in conjunction, i.e., it must be a new moon day; and
(ii) the new moon must be close to one of the nodes (*Rāhu* or *Ketu*).

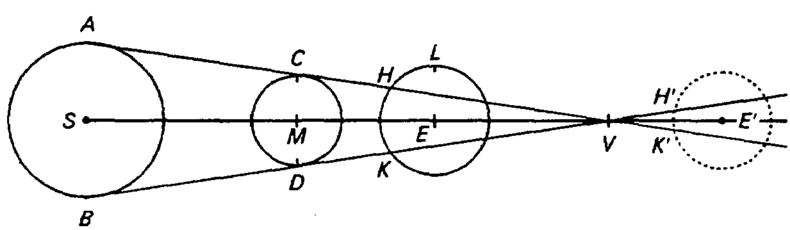

Figure 12.1 Solar eclipse

On account of the inclination of the Moon's orbit with the ecliptic (at an angle of about 5°), a solar eclipse does not occur on every new moon day. Only on those new moon days when the Moon is close to the ecliptic and hence close to one of the nodes, a solar eclipse is possible.

Since the Moon's radius is much smaller than that of the Earth's, the shadow-cone formed by the tangents to the surfaces of the Sun and the Moon can grace only a portion of the Earth. Therefore, a solar eclipse is visible only from a limited portion of the Earth's surface. In Fig.12.1,

the shadow-cone, *CVD*, of the Moon is formed by the tangents to the surfaces of the Sun and the Moon, with its vertex at *V*. This shadow-cone is called the "umbra". The "penumbra" region is obtained by drawing the internal tangents to the surfaces of the Sun and the Moon (see Fig. 12.2).

For the region on the surface of the Earth represented by the arc *HK* of the umbra (Fig. 12.1), the Sun is completely obscured by the Moon; hence, there is a *total* solar eclipse for that portion of the Earth's surface. For portions of the Earth's surface which lie in the penumbra region, such as the point *L*, the Moon covers only a part of the Sun and, hence, there will be a *partial* solar eclipse.

In fact, the total solar eclipse is possible due to the fact that the Moon's *angular diameter* at times is greater than that of the Sun (although the actual linear diameter of the Moon is quite small as compared to that of the Sun).

However, sometimes on the occasion of a solar eclipse, the angular diameter of the Moon is less than that of the Sun so that the Moon obscures only a central circular portion of the Sun leaving the outer portion of the Sun bright. Such an eclipse is called an *annular* solar eclipse. This is illustrated in Fig.12.2 where the centres of the Sun, Moon and Earth are at *S*, *M* and *E*, respectively. For the portion of the Earth's surface between H' and K', the solar eclipse is *annular*.

12.2 Angular Distance Between the Sun and the Moon at the Beginning and End of a Solar Eclipse

In Fig.12.2, the penumbra formed by the internal tangents between the surfaces of the Sun and the Moon are shown. Suppose the tangent *AB* is also tangential to the Earth's surface at *C*. Then to an observer at *C*, it is just about to enter or leave the penumbra, marking the beginning or the end of the partial phase of the solar eclipse.

Let D denote $M\hat{E}S$. We have

$$D = M\hat{E}B + B\hat{E}S \qquad \ldots \qquad (1)$$

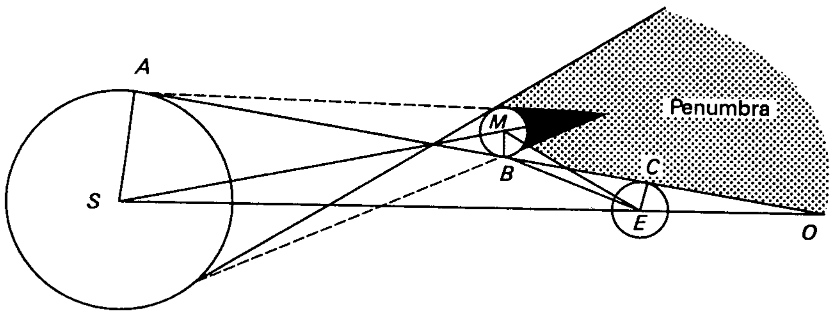

Figure 12.2 Angle MES at the beginning and end of the solar eclipse

But, during the beginning or end of the partial phase of the solar eclipse, *MB* is almost perpendicular to *EB*. Therefore,

$$M\hat{E}B = s'$$

is the Moon's angular semi-diameter, so that we have

$$D = B\hat{E}S + s'$$

Also,

$$B\hat{E}S = O\hat{B}E + E\hat{O}B \qquad \ldots \qquad (2)$$

But, $O\hat{B}E = C\hat{B}E$, the horizontal parallax of B, which is approximately equal to the horizontal parallax of the Moon, p'

i.e., $O\hat{B}E = p'$

Further, we have

$$E\hat{O}B = E\hat{O}A = A\hat{E}S - E\hat{A}C$$

or

$$E\hat{O}B = s - p$$

where s is the Sun's angular semi-diameter and p is the Sun's horizontal parallax. Therefore, from (1) and (2), we have

$$D = s + s' + p - p' \qquad \ldots \qquad (3)$$

This gives the angular distance between the Sun and the Moon with respect to the centre E of the Earth at the beginning or end of a partial solar eclipse.

Now, using the mean values

$$\begin{aligned}
&\text{Sun's semi-diameter,} & s &= 16' \\
&\text{Moon's semi-diameter,} & s' &= 15' \\
&\text{Sun's hor. parallax,} & p &= 8'' \\
&\text{Moon's hor. parallax,} & p' &= 57'
\end{aligned}$$

we get the angle $M\hat{E}S$ at the beginning or end of the partial phase of the solar eclipse given by

$$\begin{aligned}
D = M\hat{E}S &= s + s' + p - p' \\
&= 16' + 15' + 57' - 8'' \\
&= 88' - 8'' = 87'\ 52''
\end{aligned}$$

Note: As in the case of the lunar eclipse (Sec.11.3), the superior and inferior limits for the solar eclipse are respectively 18.4° and 15.4°.

12.3 Solar Eclipse According to Sūryasiddhānta

The procedure of computation of the solar eclipse is longer and more complicated than the lunar eclipse, mainly because of the effect of the parallax. In this section, we consider the procedure for the computation of a solar eclipse as given in the **Sūryasiddhānta** (*SS*). Since the positions of the Moon and *Rāhu*, in particular, are inaccurate if calculated according to *SS*, we use the true positions of the Sun, the Moon and *Rāhu* as given in the *Indian Astronomical Ephemeris* for the date considered. However, the procedure adopted is according to *SS*.

A computer program, "SSSEC" for the solar eclipse is provided in this text.

Example: The solar eclipse on 24th October, 1995, at Bangalore (according to *SS*).

I TRUE LONGITUDES AT 5:30 A.M. (IST):

1. True longitude of the Sun $= 186^\circ\ 18'\ 9''$
2. True longitude of the Moon $= 183^\circ\ 46'\ 5''$
3. True longitude of *Rāhu* $= 182^\circ\ 43'$

4. Sun's true daily motion $= 59' \, 46''$
5. Moon's true daily motion $= 14^\circ \, 1' \, 37'' = 841' \, 37''$

The latitude and the longitude of Bangalore are taken respectively as 12° N $58'$ and 77° E $35'$.

II INSTANT OF CONJUNCTION:

$$5^h.5 + \frac{\text{Diff. in true longitudes of the Sun and the Moon}}{\text{Difference in their daily motions}} \times 24^h$$

$$= \frac{5^h.5 + 186^\circ \, 18' \, 9'' - 183^\circ \, 46' \, 5''}{14^\circ \, 1' \, 37'' - 0^\circ \, 59' \, 46''} \times 24^h$$

$$= 5^h \, 30^m + 4^h \, 40^m$$

$$= 10^h \, 10^m \equiv 25^n \, 25^v \text{ after midnight}$$

Note: The *nāḍīs* and *vināḍīs* are denoted by *n* and *v* and 1 day = 60 *nāḍīs* (*Ghaṭikās*) 1 *nāḍī* = 60 *vināḍīs*.

III LONGITUDES AT THE INSTANT OF CONJUNCTION:

1. Motion of the Sun in $11^n \, 40^v = \dfrac{11^n \, 40^v \times 59' \, 46''}{60^n} = 11' \, 37''$

since $4^h \, 40^m = 11^n \, 40^v$

$\therefore$ Longitude of the Sun $= 186^\circ \, 18' \, 9'' + 11' \, 37''$
$= 186^\circ \, 29' \, 46''$

2. Moon's motion in $11^n \, 40^v = \dfrac{11^n \, 40^v \times 14^\circ \, 1' \, 37''}{60^n}$
$= 2^\circ \, 43' \, 41''$

$\therefore$ Longitude of the Moon $= 183^\circ \, 46' \, 5'' + 2^\circ \, 43' \, 41''$
$= 186^\circ \, 29' \, 46''$

3. Node's motion in $11^n \, 40^v = \dfrac{11^n \, 40^v \times (-3' \, 11'')}{60^n} = -0' \, 37''$

$\therefore$ Longitude of the node $= 182^\circ \, 43' - 0' \, 37''$
$= 182^\circ \, 42' \, 23''$

Note: Here, $4^h \, 10^m = 11^n \, 40^v$ is the time-interval for the conjunction from $5^h \, 30^m$ (IST).

IV TO FIND THE TRUE DIAMETERS OF THE SUN AND THE MOON

1. Sun's diameter $= \dfrac{6500^{y} \times 59' \, 46''}{11{,}858.75^{y}}$
$= 32' \, 46''$

2. Moon's diameter $= \dfrac{480^{y} \times 14^\circ \, 1' \, 37''}{11{,}858.75^{y}}$
$= 34' \, 4''$

In the above expressions, the Moon's mean daily linear motion in *yojanas* is about 11,858.75.

According to the *siddhāntas*, it was assumed that the Sun, the Moon and the planets move at a common linear velocity. Now, according to *SS*, the circumference of the Moon's orbit is 3.24,000 *yojanas*. The sidereal period of the Moon being 27.32167416 days, the mean linear daily motion is given by the equation: 3,24,000 / 27.32167416 = 11,858.71693 *yojanas*.

Similarly, in the case of the Sun, the circumference of the orbit is given as 43,31,500 *yojanas*. Here also, the sidereal year being 365.25875648 days, the mean linear daily motion is approximately 11,858.75 *yojanas*.

V ORIENT ECLIPTIC POINT (*LAGNA*), ETC. AT THE MOMENT OF TRUE CONJUNCTION

1. The tropical *lagna* is $264^\circ \, 30' \, 45''$

2. *Orient sine* (*Udayajyā*) : $\dfrac{R \sin \varepsilon \times R \sin (lagna)}{3350'.3}$

   where $\varepsilon = 24^\circ$, obliquity of the ecliptic (as in *SS*)
   $\therefore$ *Udayajyā* $= -1428'.375 \approx -1428'$

3. Meridian ecliptic point (*MEP*) $= 179^\circ \, 42'$

4. Meridian sine (*Madhyajyā*) $= \dfrac{R \sin \varepsilon \times R \sin (MEP)}{3438'}$
$= 7'$ in arc
$\therefore \sin^{-1} (7' / 3438') = 0^\circ \, 7' \, 22''$ in angle.
This is the declination δ of *MEP*

The meridian zenith distance (*Natāmśa*) $= \delta - \phi$
where ϕ is the latitude of the place and δ is the declination of *MEP*
Declination of *MEP* $= 0^\circ\ 7'\ 22''$ N
Latitude of the place $= 12^\circ\ 58'$ N
$\therefore$ Meridian zenith distance $= -12^\circ\ 50'\ 38''$

$$R \sin(-12^\circ\ 50'\ 38'') = -764'$$

5. The sine of the ecliptic zenith distance, *Dṛkkṣepa* :

We have $\dfrac{\text{Orient sine} \times R \text{ sine of meridian zenith distance}}{3438'}$

$$= \frac{-1428' \times -764'}{3438'} = 317'$$

Square of 317 = 100489
Deducting the above value from the square of the R sin (meridian zenith distance) and taking the square root,
we get $\sqrt{(-764)^2 - 100489} = 695'\ 10''$
That is, the sine of ecliptic zenith distance $695' = R \sin z$

6. The sine of the ecliptic altitude (*Dṛggati*)

$$= \sqrt{R^2 - R^2 \sin^2 z}$$
$$= \sqrt{(3438)^2 - (695)^2}$$
$$= 3367'$$

7. To find the divisor (*cheda*) and the Sun's parallax in longitude (*Lambana*). We have the divisor:

Cheda $= (R \sin 30^\circ)^2$ / sine of ecliptic altitude
$= (1719)^2 / 3367$
$= 878'$
i.e., divisor (*cheda*) $= 878'$
where $R \sin 30^\circ$ is given by 1719.
Sāyana Ravi at the inst. of conjn.
= Sun's sidereal longitude + *Ayanāmśa*
$= 186^\circ\ 29'\ 46'' + 23^\circ\ 47'\ 54''$
$= 210^\circ\ 17'\ 40''$
At the instant of conjunction:
Longitude of the meridian ecliptic point $= 179^\circ\ 41'\ 53''$
Longitude of the Sun (*Sāyana Ravi*) $= 210^\circ\ 17'\ 40''$

Interval in longitude (i.e.) the difference

(by adding 360°) = 329° 24′ 13″

R sine of the above is the numerator.

$$\text{Parallax in longitude } (\textit{Lambana}) = \frac{R \sin (329^\circ\, 24'\, 13'')}{\textit{Cheda}}$$

$$= -2^n\, 0^v$$

Therefore, the *corrected* instant of conjunction

$= 11^n\, 40^v - 2^n\, 0^v$

$= 9^n\, 40^v$ from 5.30 a.m. (IST)

Time of true conjunction = $25^n\, 25^v$ from the midnight

correction = $-2^n\, 0^v$

Time of app. conjunction = $23^n\, 25^v$ from the midnight.

Now, calculating the parameters again, as explained above, we get the following.

1. True Sun at the apparent conjunction : 186° 27′ 47″
2. *Rāhu* = 182° 43′ − 31″ = 182° 42′ 29″
3. True Moon : 186° 01′ 48″
4. Orient ecliptic point (*sāyana lagna*) at the moment of $23^n\, 25^v$ is 253° 39′ 02″
5. Orient sine (*Udayajyā*) $= \dfrac{R \sin \varepsilon \times R \sin (253^\circ\, 39')}{3350'.3}$

   $= -1376'.92$
6. Meridian ecliptic (*MEP*) = 166° 39′ 33″
7. Meridian sine (*Madhyajyā*) $= \dfrac{R \sin \varepsilon \times R \sin (MEP)}{3438'}$

   $= 322'\, 40''$

   Latitude of the place (ϕ) = 12° 58′ N

   Declination of *MEP* (δ) = 5° 23′ 07″ N

   Meridian zenith distance,

   Natāmśa $= \delta - \phi = -7^\circ\, 34'\, 53''$ i.e., 7° 34′ 53″ (S)

   Now, $R \sin (-7^\circ\, 34'\, 53'') = -453'\, 35''$
8. The sine of the ecliptic zenith distance (*Dṛkkṣepa*) :

   We have *Dṛkkṣepa* = 415′ 38

   $= R \sin z$

9. The sine of the ecliptic altitude (*Dṛggati*) = 3412′ 47″
10. To find the divisor (*cheda*) and the Sun's parallax in longitude (*Lambana*):
    Divisor (*cheda*) = 865′ 51″;
    Parallax in longitude (*Lambana*) = $-2^n\,44^v$
    Corrected time of apparent conjunction
    = $25^n\,25^v - 2^n\,44^v = 22^n\,41^v$ (from midnight).

Repeating the above process iteratively to get convergent values, in the *third* approximation, we obtain the following readings.

AT THE TIME OF CONJUNCTION (Approxn. 3)

True longitude of Sun	:	186° 25′ 04″
True longitude of Moon	:	185° 23′ 24″
Longitude of node	:	182° 42′ 38″
Sāyana Ravi (Trop. Sun)	:	210° 12′ 58″
Orient ecliptic point (*Lagna*)	:	249° 36′ 08″
Orient sine (*Udyajyā*)	:	−1344′.974
Meridian ecliptic point (*MEP*)	:	161° 50′ 03″
Meridian sine (*Madhyajyā*)	:	435′ 58″
Declination of the Meridian	:	7° 17′ 06″
Meridian zenith distance	:	−5° 40′ 54″
Sine of ecl. zen. dist. (*Dṛkkṣepa*)	:	313′ 14″
Sine of ecl. altitude (*Dṛggati*)	:	3423′ 42″
Divisor (*cheda*)	:	863′ 05″
Parallax in longitude (*Lambana*)	:	−2 *na.* 59 *vin.*
	=	−1H − 11M − 28S
Cor. Time of apparent conjn.	:	22 *na.* 27 *vin.*
	=	8H − 58M − 36S

AT THE TIME OF APPARENT CONJN. (after 3 iterations)

True longitude of Sun	:	186° 22′ 06″
True longitude of Moon	:	184° 41′ 37″
Longitude of node	:	182° 42′ 47″
Sāyana Ravi (Trop. Sun)	:	210° 10′ 00″
Parallax in latitude (*Nati*)	:	4′ 27″
Moon's latitude at apprnt. conjn.	:	9′ 20″

Sun's angular diameter : $32' 46''$
Moon's angular diameter : $34' 04''$
Obscurn. at apprnt. conjn. (*Grāsa*) : $19' 38''$

Eclipse is partial.

Magnitude of the eclipse : 0.5994744

CALCULATION OF THE HALF-DURATION OF THE ECLIPSE

The square of the sum of the semi-diameters : $(33' 25'')^2$
$= 1116' 40''$
The square of the Moon's latitude : $(13' 47'')^2$
$= 189' 58''$
Subtracting : $926' 42''$

Here, the Moon's *apparent* latitude at the apparent conjunction is considered.

Square root of the above diff. $= \sqrt{926' 42''}$
$= 30' 26''.5$

Half-interval $= (60^n \times 30' 26''.5)$ / Diff. in true daily motions of the Sun and Moon
$= (60^n \times 30' 26''.5) / 783' 51'' = 2^n 19'$

Beginning of the eclipse
= Time of apparent conjunction – $(2^n 19^v)$
$= 22^n 27^v - 2^n 19^v$
$= 20^n 08^v \equiv 8^h 03^m 12^s$

End of the eclipse
$= 22^n 27^v + 2^n 19^v$
$= 24^n 46^v \equiv 9^h 54^m 24^s$

To obtain more accurate values for the beginning and the end of the eclipse, further iterations of the above procedure must be carried out.

After a few iterations, the values converge reasonably to yield the following:

First half-duration of eclipse : 2 *na*. 25 *vin*. = 0H – 58M – 1S
Second half-duration of eclipse: 2 *na*. 15 *vin*. = 0H – 53M – 49S

Summary of the solar eclipse on 24 / 10 / 1995 at Bangalore (IST)

Beginning of the eclipse	: 20 *na*. 01 *vin*.	8H – 00M – 35S
Middle of the eclipse	: 22 *na*. 27 *vin*.	8H – 58M – 36S
End of the eclipse	: 24 *na*. 41 *vin*.	9H – 52M – 25S

12.4 SAROS AND METONIC CYCLE

The interval between two successive passages of the Sun through a node is 346.62 days, and 19 such intervals amount to 6585.8 days. The mean length of a synodic month (or lunation) is about 29.5306 days and, therefore, 223 lunations are equivalent to 6585.3 days. We thus have the approximate relation:

223 lunations = 19 revolutions of the Sun

with respect to a node (say, *Rāhu*). This period of 6585 days is called saros and is equivalent to about 18 years and 11 days. Eclipses are generally repeated once in a saros period i.e., 18 years, 11 days.

Example: There was a (total) solar eclipse on October 24th, 1995. After a saros cycle, there will again be a solar eclipse on November 3rd, 2013.

Similar to the saros, there is another period called metonic cycle of about 19 years. We notice that 235 lunations are equivalent to 19 years of 365.25 days, amounting to about 6939.7 days. New moons and full moons repeat once in a metonic cycle.

12.5 CONCLUSION

By a similar procedure as in section 12.3, the beginning and the end of the maximum obscurity (or totality, as the case may be) are determined. The computations of the solar eclipse are quite complicated. This is largely due to the pronounced effect of the parallax. The later Indian astronomers, particularly those of Kerala, periodically gave improved elements as corrections (*samskāra*) based on continuous observation of a large number of eclipses during their lifetimes. Especially Parameśvara (c.1360–1455 AD) and Nīlakaṇṭha Somayāji (1444–1545 AD), two Kerala astronomers, are famous for their careful observations and corrections based on them.

The procedures adopted by the traditional Siddhāntic texts are generally correct. The errors in the timings can be corrected by adopting modern values for the parameters. The suggested improvements, based on modern astronomy, in the traditional computations have been worked out by the author of this book and will be presented subsequently.

Computer Programs

```
10  CLS:KEY OFF: REM * PROGRAM: "SSRAMOON" *
20  PRINT TAB(23);"******************************************"
30  PRINT TAB(23);"*            SURYASIDDHANTA               *"
40  PRINT TAB(23);"* POSITIONS OF SUN, MOON AND RAHU *"
50  PRINT TAB(23);"******************************************"
60  PI=3.141592653589793#
70  LOCATE 5,16:PRINT "(CHRISTIAN) DATE : ":LOCATE 5,40:LINE INPUT
    "YEAR: ";YE$:LOCATE 5,53:LINE INPUT "MONTH: ";MO$:LOCATE
    5,65:LINE INPUT "DATE: ";DA$:Y=VAL(YE$):MM=VAL(MO$)
    :D1=VAL(DA$)
80  LOCATE 6,16:PRINT "TIME (AFTER MIDNIGHT): ";:LOCATE 6,40:LINE
    INPUT "HOURS: ";HR$:LOCATE 6,55:LINE INPUT "MINUTES: ";MIN$
    :H1=VAL(HR$) :MI=VAL(MIN$)
90  LOCATE 7,16:PRINT "NAME OF THE PLACE: ";:LOCATE 7,40
    :LINE INPUT PLACE$
100 LOCATE 8,16:PRINT "LONGITUDE (-ve for West): ";:LOCATE 8,45
    :LINE INPUT "DEG: ";LD$:LOCATE 8,60:LINE INPUT "MIN: ";LM$
110 LOCATE 9,16:PRINT "LATITUDE (-ve for South): ";:LOCATE 9,46
    :LINE INPUT "DEG: ";PD$:LOCATE 9,60:LINE INPUT "MIN: ";PM$
120 LD=VAL(LD$):LM=VAL(LM$):PD=VAL(PD$):PM=VAL(PM$)
130 IF LD<0 THEN LAM=LD-LM/60:GOTO 150
140 LAM=LD+LM/60
150 IF PD<0 THEN PHI=PD-PM/60:GOTO 170
160 PHI=PD+PM/60
170 REM *** UJJAYINI:LONG.75.75E, LAT.23.18N ***
180 ULAM=75.75-LAM:REM ** LONG. w.r.t. UJJAYINI ***
190 TC = INT((Y-1900)/100)
200 T = Y-100*INT(Y/100)
210 IF TC<-4THEN E=13
220 IF TC=-4 AND Y<1582 THEN E=13
230 IF TC=-4 AND Y>1582 THEN E=3
240 IF TC>-4 AND TC<=0 THEN E=-TC
250 IF TC>0 THEN E=-(TC-1)
```

```
260 Q=-(T MOD 4)
270 DD=0
280 DATA 0,31,28,31,30,31,30,31,31,30,31,30
290 RESTORE 280:FOR I=1 TO MM
300 READ X
310 DD=DD+X
320 NEXT I
330 IF (Y>1600) AND (Y/100=INT(Y/100)) AND (Y/400<>INT(Y/400))
    THEN GOTO 350
340 IF Y/4=INT(Y/4) AND (MM=1 OR MM=2) THEN DD=DD-1
350 JJ=(TC*100+T)*365.25+DD+D1+E+(Q/4)+2415020!
360 LET KALI = JJ-588466!
370 WD=JJ-7*INT(JJ/7)
380 RESTORE 410
390 FOR F=0 TO WD
400 READ X$
410 DATA MONDAY,TUESDAY,WEDNESDAY,THURSDAY,FRIDAY,SATURDAY,
    SUNDAY
420 NEXT F
430 PRINT TAB(22); "******** WEEK DAY: ";X$; "*********"
440 PRINT "KALI YUGA DAYS ELAPSED (EPOCH:Feb. 17/18,3102 BC): ";
    KALI
450 PRINT "********************* RAVI SPHUTA***********************"
460 REM ** 4 320 000 REVOLNS IN 1 577 917 828 CIVIL DAYS **
470 DAILY#= 2.737785151635919D-03:REM ** REVOLN **
480 MRAVI#=KALI*DAILY# :REM ** REVOLNS SINCE KALI EPOCH **
490 REV=INT(MRAVI#):PRINT TAB(15) "REVOLNS SINCE KALI EPOCH: ";
    REV
500 MRAVI = 360*(MRAVI#-REV) :REM ** DEGREES **
510 PRINT "MEAN RAVI AT UJJAYINI MIDNIGHT: ";
520 L=MRAVI:GOSUB 1550
530 REM *** DESHANTARA COR. ** SUN'S DAILY MOTION:59'8'10'"10.4"" **
540 DAILY#=.9856026545889309#:REM * DEG *
550 GOSUB 1460
560 KAALA=(H1+MI/60)*DAILY#/24
570 PRINT "MOTION FOR";H1; "HRS";MI; MIN: ";
580 L=KAALA:GOSUB 1550
590 PRINT "MEAN RAVI AT THE GIVEN LOCAL TIME";
600 MRAVI=MRAVI+DESH+KAALA:L=MRAVI:GOSUB 1550
```

```
610  REM ****** SUN'S MANDOCCA******
620  CIVIL=1577917828#:REM * 387 REVNS IN 1577917828*1000 CIVIL DAYS*
630  SMA=77.13+(KALI*387*360/(CIVIL*1000))
640  PMA1=SMA-MRAVI:REM *** RAVI'S MANDA KENDRA ***
650  IF PMA1<0 THEN PMA1=PMA1 +360
660  K1=14/(2*PI):K2=20/(60*2*PI)
670  PMA=SMA : SMK=PMA1 :REM * SUN'S ANOMALY *
680  GOSUB 1610
690  SEQ=PEQ
700  TRAVI=MRAVI+SEQ
710  IF TRAVI<0 THEN TRAVI=TRAVI+360
720  IF TRAVI>360 THEN TRAVI=TRAVI-360
730  PRINT "-——————————————————————————"
740  PRINT "TRUE RAVI: ";
750  L=TRAVI:GOSUB 1550
760  PRINT "-——————————————————————————"
770  LOCATE 23,60:PRINT "PRESS ANY KEY TWICE"
780  A$=INPUT$(2)
790  PRINT:PRINT
800  PRINT "**********CHANDRA SPHUTA**********"
810  REM ** NO.OF REVNS:57 753 336 FOR 1 577 917 828 CIVIL DAYS **
820  REM *** DAILY MEAN MOTION:790'34"521'"3.8""  ***
830  DAILY#= 3.660097818477782D-02:REM ** REVOLN
840  MOON#=KALI*DAILY#
850  REV=INT(MOON#):PRINT TAB(15) "REVOLNS SINCE KALI EPOCH: ";
     REV
860  MOON = 360*(MOON#-REV)
870  PRINT "MEAN MOON AT UJJAYINI MIDNIGHT: ";
880  L=MOON:GOSUB 1550
890  DAILY#=360*DAILY#:GOSUB 1480
900  KAALA=(H1+MI/60)*DAILY/24
910  PRINT "MOTION FOR";H1; "HRS";MI; " MIN: ";
920  L=KAALA:GOSUB 1550
930  PRINT "MEAN MOON AT GIVEN TIME AT";PLACE$;
940  MOON=MOON+DESH+KAALA:L=MOON:GOSUB 1550
950  MMA =.25+(KALI*488203!/CIVIL):REM ** MOON'S MANDOCCA **
960  MMA = 360*(MMA-INT(MMA))
970  PMA1 = MMA-MOON
980  IF PMA1<0 THEN PMA1=PMA1+360
```

```
990  K1=32/(2*PI):K2=20/(60*2*PI)
1000 PMA=MMA:MMK=PMA1:REM * MMK:MOON'S ANOMALY *
1010 GOSUB 1610
1020 TMOON=MOON+PEQ
1030 REM *** BHUJANTARA CORRECTION ***
1040 BHUJ=SEQ/27
1050 PRINT "BHUJANTARA CORRECTION: ";
1060 IF BHUJ>0 THEN PRINT TAB(39); "-";
1070 IF BHUJ<0 THEN PRINT TAB(39); "+";
1080 L=BHUJ:GOSUB 1550
1090 TMOON=TMOON+BHUJ
1100 IF TMOON<0 THEN TMOON=TMOON+360
1110 IF TMOON>360 THEN TMOON=TMOON-360
1120 PRINT "——————————————————————"
1130 PRINT "TRUE MOON: ";
1140 L=TMOON:GOSUB 1550
1150 PRINT "——————————————————————"
1160 PRINT "***********RAHU SPHUTA**********"
1170 REM *NO. OF REVNS. -232 238 REVOLNS IN 1 577 917 828 CIVIL DAYS*
1180 DAILY#=-1.4718003426982D-04:REM ** REVOLN **
1190 MRAHU=.5+KALI*DAILY# :REM ** REVOLNS * 180 DEG AT EPOCH **
1200 REV=INT(MRAHU)+ 1:PRINT TAB(15)"REVLNS SINCE KALI EPOCH: ";
     REV
1210 MRAHU=360*(MRAHU-REV)
1220 MRAHU=360+MRAHU
1230 PRINT "——————————————————————"
1240 PRINT "MEAN RAHU AT UJJAYINI MIDNIGHT: ";
1250 L=MRAHU:GOSUB 1550
1260 REM *** DAILY MOTION: 3'10.745" = 0.05298481616778111 DEG ***
1270 DAILY#=-1.4718003426982D-04*360:GOSUB 1480:REM ** DEG
1280 KAALA=(H1+MI/60)*DAILY#/24
1290 PRINT "MOTION FOR";H1; "HRS";MI; "MIN: ";
1300 IF KAALA<0 THEN PRINT TAB(39) "-";
1310 L=KAALA:GOSUB 1550
1320 PRINT "——————————————————————————————"
1330 PRINT "MEAN RAHU AT GIVEN TIME AT";PLACE$;
1340 MRAHU=MRAHU+DESH+KAALA:L=MRAHU:GOSUB 1550
1350 PRINT "——————————————————————————————"
1360 VRK=TMOON-TRAVI:IF VRK<0 THEN VRK=VRK+360
```

```
1370 IF VRK<12 OR VRK>348 THEN PRINT TAB(34); "NEW MOON DAY" :ZZ=2
1380 IF VRK>168 AND VRK<192 THEN PRINT TAB(34); "FULL MOON DAY"
     :ZZ=1
1390 IF VRK>12 AND VRK<168 THEN PRINT TAB(28); "NOT NEW OR FULL
     MOON DAY"
1400 LOCATE 24, 10:INPUT "DO YOU WANT ECLIPSE OR PLANETS
     COMPUTATIONS(E/P)";A$
1410 IF ZZ<>1 AND ZZ<>2 AND (A$= "E" OR A$= "e")
     THEN PRINT TAB(30) "ECLIPSE NOT POSSIBLE":END
1420 IF ZZ=1 AND A$= "E" OR A$= "e" THEN T0=H1+MI/60 :CHAIN
     "SSLEC",,ALL
1430 IF ZZ=2 AND (A$= "E" OR A$= "e") THEN T0=H1+MI/60 :CHAIN
     "SSSEC",,ALL
1440 IF A$= "P" OR A$= "p" THEN CHAIN "SSPLA",,ALL
1450 IF A$<> "E" AND A$<> "e" AND A$<> "P" AND A$<> "p" THEN END
1460 REM *** DESHANTARA CORRECTION ***
1470 DESH=ULAM*DAILY#/360
1480 PRINT "DESHANTARA CORRECTION: ";
1490 IF DAILY#<0 AND ULAM>0 THEN PRINT TAB(39) "-";:GOTO 1530
1500 IF DAILY#<0 AND ULAM<0THEN PRINT TAB(39) "+";:GOTO 1530
1510 IF ULAM>0 THEN PRINT TAB(39) "+";
1520 IF ULAM<0 THEN PRINT TAB(39) "-";
1530 L=DESH:GOSUB 1550
1540 RETURN
1550 IF L<0 THEN L=ABS(L)
1560 DEG=INT(L):MIN=(L-DEG)*60:SEC=INT((MIN-INT(MIN))*60+.5)
1570 IF DEG>=360 THEN DEG=DEG-360
1580 IF SEC=60 THEN SEC=0:MIN=MIN+1: IF MIN=60 THEN MIN=0
     :DEG=DEG+1: IF DEG>=360 THEN DEG=DEG-360
1590 PRINT TAB(40);DEG; "°";INT(MIN); "'";SEC;"""
1600 RETURN
1610 REM *** EQUATION OF CENTRE ***
1620 IF Z=0 THEN PRINT "MANDOCCA: ";:L=PMA:GOSUB 1550
1630 PRINT "MANDA ANOMALY: ";
1640 L=PMA1:GOSUB 1550
1650 PRINT "MANDA EQUATION : ";
1660 PMA1=PMA1*PI/180: REM ** RADIANS **
1670 SN=SIN(PMA1)
1680 PEQ=(K1-K2*ABS(SN))*SN:REM ** PLANET'S EQN. OF CENTRE **
1690 IF PMA1<PI THEN PRINT TAB(39) "+";
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1700 IF PMA1>PI THEN PRINT TAB (39) "-";
1710 L=PEQ:GOSUB 1550
1720 RETURN
```

```
10  CLS:KEY OFF: REM * PROGRAM "SSLEC"
20  PRINT TAB(18) "********************************************************"
30  PRINT TAB(18) "LUNAR ECLIPSE ACCORDING TO SURYA SIDDHANTA"
40  PRINT TAB(18) "********************************************************"
50  PI=3.141592653589793#
60  REM * TRUE RAVI,MOON,RAHU AT ISHTAKAALA T0, IN GHATIS *
80  PRINT "AT";GH; "GH";VIG; "VIG. FROM MIDNIGHT ON";D1; "/"; MM; "/";Y
90  PRINT "TRUE RAVI: ";TRAVI; "TRUE MOON: ";TMOON; "RAHU: "; MRAHU
100 REM * SMK:SUN'S ANOMALY; MMK:MOON'S ANOMALY *
110 REM * SUN'S MEAN DAILY MOTION: 59'8.17"; MOON'S: 790'34.866" *
120 SDM=59.136*(1-(14/360)*COS(PI*SMK/180)):REM * SUN'S TRUE DAILY
    MOTION *
130 MDM=790.581#-(31/360)*783.898*COS(PI*MMK/180):REM * MOON'S
    TRUE DAILY MOTION *
140 NDM=191/60: REM * RAHU'S MEAN DAILY MOTION (MIN) *
150 REM * 783.898=MEAN DAILY MOTION OF (MOON-MOON'S APOGEE)*
160 SOPP=(TRAVI-TMOON)+180:IF SOPP<0 THEN SOPP=SOPP+360
170 IF SOPP>360 THEN SOPP=SOPP-360
180 IF SOPP<0 THEN SOPP=SOPP+360
190 PRINT "MOON'S DISTANCE FROM OPPN.: ";:L=SOPP:GOSUB 990
200 OPPT=SOPP*60/((MDM-SDM)/60)
220 PRINT "INSTANT OF OPPN. AFTER MIDNIGHT: ";:H=(T0+OPPT)*2/5
    :GOSUB 1040
230 PRINT TAB(23) "*** AT THE INSTANT OF OPPOSITION ***"
240 OPSUN=TRAVI+OPPT*SDM/(60*60)
250 PRINT "TRUE RAVI AT OPPN.: ";:L=OPSUN:GOSUB 990
260 OPMOON=TMOON+OPPT*MDM/(60*60)
270 PRINT "TRUE CHANDRA AT OPPN.: ";:L=OPMOON:GOSUB 990
280 OPNODE=MRAHU-190.745*OPPT/(3600*60)
290 PRINT "RAHU AT OPPN.: ";:L=OPNODE:GOSUB 990
```

```
300 SCDIA=SDM*6500/59.136:REM * SUN'S COR.DIAMETER (YOJANAS) *
310 SDIAY=SCDIA*4320000!/57753336# :REM * SUN'S DIAMETER IN YOJANAS *
320 SDIA=SDIAY/15: REM * SUN'S DIAMETER IN ARC (MIN) *
330 PRINT "SUN'S ANGULAR DIAMETER: ";:M=SDIA:GOSUB 1090
340 MDIAY=MDM*480/790.581: REM * MOON'S DIAMETER IN YOJANAS *
350 MDIA=MDIAY/15: REM * MOON'S DIAMETER IN ARC (MIN) *
360 PRINT "MOON'S ANGULAR DIAMETER: ";:M=MDIA:GOSUB 1090
370 ECDIA=MDM*1600/790.581 :REM * EARTH'S COR.DIAMETER (YOJANAS) *
380 SCDEDIA=SCDIA - 1600 :REM * SUN'S COR.DIA - EARTH'S MEAN DIA *
390 SHDIAY = ECDIA - SCDEDIA*480/6500 :REM*SHADOW'S DIAMETER
    (YOJANAS) *
400 SHDIA = SHDIAY/15: REM * SHADOW'S DIAMETER IN ARC (MIN) *
410 PRINT "SHADOW'S ANGULAR DIAMETER: ";:M=SHDIA:GOSUB 1090
420  MLAT=270*SIN(PI*(OPMOON-OPNODE)/180):MLATI=MLAT
430 PRINT "MOON'S LATITUDE AT OPPN.: ";:IF MLAT<0 THEN PRINT
    TAB(34); "-";
440 M=MLAT:GOSUB 1090
450 GRASA=.5*(MDIA+SHDIA)-MLAT
460 PRINT "MOON'S OBSCURED PORTION (GRASA): ";:M=GRASA
    :GOSUB 1090
470 IFGRASA>0 AND GRASA<MDIA THEN PRINT TAB(29) "LUNAR ECLIPSE
    IS PARTIAL":ZZ=1
480 IF GRASA<0 THEN PRINT TAB(29) "LUNAR ECLIPSE NOT POSSIBLE" :END
490 IF GRASA>=MDIA THEN PRINT TAB(29) "LUNAR ECLIPSE IS TOTAL"
500 PRINT TAB(60) "PRESS ANY KEY":A$=INPUT$(1)
510 CLS:REM * HALF-DURATIONS OF ECLIPSE AND TOTALITY *
520 HDUR=60*SQR(((SHDIA+MDIA)/2)^2-MLAT^2)/(MDM-SDM)
530 PRINT "HALF-DURATION OF THE ECLIPSE: ";TAB(35);HDUR; "NAADIS"
540 IF ZZ=1 THEN GOTO 570
550 HDURT=60*SQR(((SHDIA-MDIA)/2)^2 - MLAT^2)/(MDM-SDM)
560 PRINT "HALF-DURATION OF THE TOTALITY: ";TAB(35);HDURT; "NAADIS"
570 REM * AT THE END OF THE ECLIPSE *
580 TMOON=OPMOON+HDUR*MDM/3600
    :MRAHU=OPNODE-HDUR*NDM/3600: REM *DEG*
590 MLAT=270*SIN(PI*(TMOON-MRAHU)/180)
    :IF MLAT<0 THEN PRINT TAB(34) "-";
600 PRINT "MOON'S LATITUDE AT THE END: ";:M=MLAT:GOSUB 1090
610 HDUR2=60*SQR(((SHDIA+MDIA)/2)^2-MLAT^2)/(MDM-SDM)
620 PRINT "COR.SECOND HALF-DURN.OF ECLIPSE: ";TAB(35);HDUR2;
    "NAADIS"
```

```
630 REM * AT THE BEGINNING OF THE ECLIPSE *
640 TMOON=OPMOON-HDUR*MDM/3600
    :MRAHU=OPNODE+HDUR*NDM/3600:REM *DEG*
650 MLAT=270*SIN(PI*(TMOON-MRAHU)/180)
    :IF MLAT<0 THEN PRINT TAB(34) "-";
660 PRINT "MOON'S LAT.AT BEGIN OF ECLIPSE: ";:M=MLAT :GOSUB 1090
670 HDUR1=60*SQR(((SHDIA+MDIA)/2)^2-MLAT^2)/(MDM-SDM)
680 PRINT "COR.FIRST HALF-DURN.OF ECLIPSE: ";TAB(35);HDUR1; "NAADIS"
690 IF ZZ=1 THEN GOTO 820
700 REM * AT THE END OF THE TOTALITY *
710 TMOON=OPMOON+HDURT*MDM/3600
    :MRAHU=OPNODE-HDURT*NDM/3600:REM *DEG*
720 MLAT=270*SIN(PI*(TMOON-MRAHU)/180)
    :IF MLAT<0 THEN PRINT TAB(34) "-";
730 PRINT "MOON'S LAT. AT END OF TOTALITY: ";:M=MLAT :GOSUB 1090
740 HDURT2=60*SQR(((SHDIA-MDIA)/2)^2-MLAT^2)/(MDM-SDM)
750 PRINT "COR.SECOND HALF-DURN.OF TOTALITY: ";TAB(35); HDURT2;
    "NAADIS"
760 REM * AT THE BEGINNING OF THE TOTALITY *
770 TMOON=OPMOON-HDURT*MDM/3600
    :MRAHU=OPNODE+HDURT*NDM/3600:REM *DEG*
780 MLAT=270*SIN(PI*(TMOON-MRAHU)/180)
    :IF MLAT<0 THEN PRINT TAB(34) "-";
790 PRINT "MOON'S LAT.AT BEGIN.OF TOTALITY: ";:M=MLAT :GOSUB 1090
800 HDURT1=60*SQR(((SHDIA-MDIA)/2)^2-MLAT^2)/(MDM-SDM)
810 PRINT "COR.FIRST HALF-DURN.OF TOTALITY: ";TAB(35);HDURT1;
    "NAADIS"
820 PRINT TAB(24) "****************************************"
830 PRINT TAB(24) "* SUMMARY OF THE LUNAR ECLIPSE *"
840 PRINT TAB(24) "****************************************"
850 BEG=(T0+OPPT-HDUR1)*2/5 :REM * BEGINNING OF ECLIPSE IN HRS *
860 PRINT "BEGINNING OF THE ECLIPSE: ";:H=BEG:GOSUB 1040
870 IF ZZ=1 THEN GOTO 900
880 BEGT=(T0+OPPT-HDURT1)*2/5
    :REM * BEGINNING OF TOTALITY IN HRS *
890 PRINT "BEGINNING OF THE TOTALITY: ";:H=BEGT:GOSUB 1040
900 MID=(T0+OPPT)*2/5:REM * MIDDLE OF THE ECLIPSE IN HRS *
910 PRINT "MIDDLE OF THE ECLIPSE: ";:H=MID:GOSUB 1040
920 IF ZZ=1 THEN GOTO 950
930 ENDT=(T0+OPPT+HDURT2)*2/5:REM * END OF TOTALITY IN HRS *
940 PRINT "END OF THE TOTALITY: ";:H=ENDT:GOSUB 1040
```

```
950  ENDE=(T0+OPPT+HDUR2)*2/5 :REM * END OF ECLIPSE IN HRS *
960  PRINT "END OF THE ECLIPSE: ";:H=ENDE:GOSUB 1040
970  END
980  REM * CONVERSION INTO DEG,MIN,SEC OF ARC*
990  IF L<0 THEN L=ABS(L)
1000 DEG=INT(L):MINT=(L-DEG)*60:MIN=INT(MINT)
     : SEC=INT((MINT-MIN)* 60+.5)
1010 IF SEC=60 THEN SEC=0:MIN=MIN+1:IF MIN=60 THEN MIN=0
     :DEG=DEG+1
1020 PRINT TAB(35);DEG; "°";MIN; "'" SEC; """
1030 RETURN
1040 REM * CONVERSION TO HRS,MIN,SEC OF TIME *
1050 HRS=INT(H):MNT=(H-HRS)*60:MIN=INT(MNT)
     :SEC=INT((MNT-MIN)*60+.5)
1060 IF SEC=60 THEN SEC=0:MIN=MIN+1:IF MIN=60
     THEN MIN=0:HRS=HRS+1
1070 PRINT TAB(35);HRS; "H-";MIN; "M-";SEC; "S"
1080 RETURN
1090 REM * CONVERSION TO MIN AND SEC OF ARC *
1100 MIN=INT(M):SEC=INT((M-MIN)*60+.5) :IF SEC=60
     THEN SEC=0:MIN=MIN+1
1110 PRINT TAB(35);MIN; "'";SEC; """
1120 RETURN
```

```
10   CLS:REM * PROGRAM "SSSEC"
20   PRINT TAB(23); "******************************"
30   PRINT TAB(23); "*       SOLAR ECLIPSE        *"
40   PRINT TAB(23); "*       ACCORDING TO         *"
50   PRINT TAB(23); "*     SURYA SIDDHANTA        *"
60   PRINT TAB(23); "******************************"
70   PRINT:PI=3.14159256#:DTR=PI/180: RTD=1/DTR :REM * DEG TO RAD &
     RAD TO DEG *
140  REM ** True Sun (TRAVI),True Moon (TMOON), Node (MRAHU) at T0 HRS **
150  PRINT:PRINT TAB(25) "AT";T0; "HRS ON";D1; "/";MM; "/";Y
160  T=(Y-1900+(MM-1)/12+D1/365)/100
     :REM * JULIAN CENTURIES SINCE 1/ 1/1900 *
```

```
170 AYA=22.4604222#+(1.3960416#*T)+(1.111/3600)*T*T+ (.0001 *T*T*T/3600)
180 PRINT:PRINT TAB(25); "AYANAMSA: ";:L=AYA:GOSUB 2130
190 PRINT:PRINT "*** TRUE SUN: ";TRAVI; "TRUE MOON: ";TMOON;
    "NODE: "; MRAHU
200 PRINT
210 REM ** SMK:Sun's anomaly; MMK:Moon's anomaly **
220 SDM=59.13333*(1-14*COS(SMK*PI/180)/360) :REM * SUN'S TRUE DAILY
    MOTION *
230 PRINT "SUN'S TRUE DAILY MOTION: ";:M=SDM:GOSUB 2080
240 MDM=790.5666-31*783.9*COS(MMK*PI/180)/360 :REM * MOON'S TRUE
    DAILY MOTION *
250 REM ** 783.9 = MEAN DAILY MOTION OF (MOON - MOON'S APOGEE) **
260 PRINT:PRINT "MOON'S TRUE DAILY MOTION: ";:M=MDM :GOSUB 2080
270 SCON=(TRAVI-TMOON):IF SCON<0 THEN SCON=SCON+360
280 PRINT:PRINT "MOON'S DISTANCE FROM CONJN.: ";:L=SCON :GOSUB 2130
290 TCON=SCON*24/((MDM-SDM)/60)
300 H1=T0+TCON:H(0)=H1
310 PRINT:PRINT "TIME OF CONJN.AFTER MIDNIGHT";:N=H1*5/2
    :GOSUB 2200
320 H=H1:GOSUB 1120
330 PRINT:PRINT TAB(60) "PRESS ANY KEY":A$=INPUT$(1)
350 FOR I= 1 TO 20
360 CLS:PRINT TAB(15) "* AT THE TIME OF CONJUNCTION (APPROXN "; I;") *"
370 IF I<>I0 THEN GOTO 390
380 CLS:PRINT TAB(12) "* AT THE TIME OF APPARENT CONJN. (AFTER";
    I; "ITERATIONS *"
390 CSUN1=TRAVI+TCON*SDM/(60*24)
400 PRINT "TRUE LONG. OF SUN: ";:L=CSUN1:GOSUB 2130
410 CMOON1=TMOON+TCON*MDM/(60*24)
420 PRINT "TRUE LONG. OF MOON: ";:L=CMOON1:GOSUB 2130
430 CNODE1=MRAHU-190.7*TCON/(3600*24)
440 PRINT "LONG. OF NODE: ";:L=CNODE1:GOSUB 2130
450 TSUN1=CSUN1+AYA:IF TSUN1>360 THEN TSUN1=TSUN1-360
460 IF TSUN1<0 THEN TSUN1=TSUN1+360
470 PRINT:PRINT "SAYANA RAVI (TROP.SUN): ";:L=TSUN1:GOSUB 2130
480 IF I=I0 THEN GOTO 840
490 PRINT "ORIENT ECLIPTIC PT.(LAGNA): ";:T=H(I- 1):GOSUB 1660
500 OREC=TLAG: L=OREC:GOSUB 2130
510 ORSIN = 3438*SIN(24*DTR)*SIN(OREC*DTR)/COS(PHI*DTR)
    :REM * ORIENT SINE: UDAYAJYA*
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```
520 PRINT "ORIENT SINE (UDAYA JYA): ";TAB(35);ORSIN; "'"
530 PRINT:PRINT "MERIDIAN ECLIPTIC POINT (M.C.): ";:L=MC :GOSUB 2130
540 MERSIN=3438*SIN(24*DTR)*SIN(MC*DTR): REM * MERIDIAN SINE *
550 PRINT "MERIDIAN SINE (MADHYA JYA): ";:M=MERSIN:GOSUB 2080
560 X=MERSIN /3438:GOSUB 880:REM * DECLINATION OF MC (in Deg)*
570 PRINT "DECLINATION OF THE MERIDIAN: ";
580 IF DEC<0 THEN PRINT TAB(34); "-";
590 L=DEC:GOSUB 2130
600 MERZEN = DEC - PHI: REM * PHI=PD+PM/60: LAT OF THE PLACE*
610 PRINT "MERIDIAN ZENITH DISTANCE: ";:IF MERZEN<0
    THEN PRINT TAB(34); "-";
620 L=MERZEN:GOSUB 2130
630 SNMERZ=3438*SIN(MERZEN*DTR)
640 OSMZ = ORSIN * SNMERZ/3438
650 DRKSHEPA = SQR(SNMERZ^2 - OSMZ^2)
660 PRINT "SINE OF ECL.ZEN.DIST.(DRKKSHEPA): "; :M=DRKSHEPA
    :GOSUB 2080
670 SNECALT=SQR(3438^2-DRKSHEPA^2)
680 PRINT "SINE OF ECL.ALTITUDE (DRGGATI): ";:M=SNECALT :GOSUB 2080
690 CHEDA=1719^2/SNECALT:REM * RSIN(30 Deg)=1719 *
700 PRINT "DIVISOR (CHEDA): ";:M=CHEDA:GOSUB 2080
710 LAMBANA=3438*(SIN((MC-TSUN1)*DTR))/CHEDA
720 PRINT "(PARALLAX IN LONG.) LAMBANA: ";
730 IF LAMBANA<0 THEN PRINT TAB(34); "-";
740 N=LAMBANA:GOSUB 2200
750 H=LAMBANA*2/5:IF H<0 THEN PRINT "-";
760 GOSUB 1120
770 H(I)=H1+LAMBANA*2/5
780 PRINT:PRINT "COR.TIME OF APPARENT CONJN.: ";:N=H(I)*5/2
    :GOSUB 2200
790 H=H(I):GOSUB 1120
795 PRINT "H(";I; ")=";H(I)
800 TCON=TCON+2*LAMBANA/5
810 PRINT:PRINT TAB(60); "PRESS ANY KEY":A$=INPUT$(1)
820 IF I=20 OR ABS(H(I)-H(I-1))<1 THEN I0=I: GOTO 380
830 NEXT I
840 NATI=(731.45/15)*DRKSHEPA/3438
850 PRINT "PARALLAX IN LATITUDE (NATI): "; :IF NATI<0 THEN TAB(34); "-";
860 M=NATI:GOSUB 2080
```

```
870  GOTO 900
880  DEC=RTD*ATN(X/SQR(1-X*X))
890  RETURN
900  MLAT1=270*SIN(DTR*(CMOON1-CNODE1))
910  PRINT "MOON'S LATITUDE AT APPNT.CONJN.: ";:IF MLAT1<0
     THEN PRINT TAB(34) "-";
920  M=MLAT1:GOSUB 2080
930  APLAT=MLAT1+NATI
940  PRINT "MOON'S APPARENT LAT. AT CONJN.: ";:IF APLAT<0
     THEN PRINT TAB(34) "-";
950  M=APLAT:GOSUB 2080
960  SCDIA=SDM*6500/59.136:REM * SUN'S COR.DIAMETER (YOJANAS) *
970  SDIAY=SCDIA*4320000!/57753336#
     :REM * SUN'S DIAMETER IN YOJANAS*
980  SDIA=SDIAY/15: REM * SUN'S DIAMETER IN ARC (MIN) *
990  PRINT:PRINT "SUN'S ANGULAR DIAMETER: ";:M=SDIA:GOSUB 2080
1000 MDIAY=MDM*480/790.581: REM * MOON'S DIAMETER IN YOJANAS *
1010 MDIA=MDIAY/15: REM * MOON'S DIAMETER IN ARC (MIN) *
1020 PRINT "MOON'S ANGULAR DIAMETER: ";:M=MDIA:GOSUB 2080
1030 OBS=.5*(SDIA+MDIA)-ABS(APLAT)
1040 PRINT:PRINT "OBSCURN.AT APPRNT.CONJN (GRAASA): ";:M=OBS
     :GOSUB 2080
1050 PRINT
1060 IF OBS<0 THEN PRINT TAB(30) "ECLIPSE NOT VISIBLE":END
1070 IF OBS<SDIA THEN PRINT TAB(31) "ECLIPSE IS PARTIAL":ZZ=1
1080 IF OBS>=SDIA THEN PRINT TAB(31) "ECLIPSE IS TOTAL"
1090 PRINT:PRINT TAB(20) "MAGNITUDE OF THE ECLIPSE: ";OBS/SDIA
1100 LOCATE 22,60:PRINT "<PRESS ANY KEY>":A$=INPUT $(1)
1110 PRINT:IF ZZ=1 THEN GOTO 1170
1120 REM ** CONVERSION INTO HRS, MIN, SEC **
1130  H=ABS(H):HRS=INT(H):M=60*(H-HRS):MIN=INT(M)
     :SEC=INT((M-MIN)* 60)
1140 PRINT TAB(54) HRS; "H-";MIN; "M-";SEC; "S"
1150 RETURN
1160 REM * HALF-DURATIONS *
1170 DBET=SIN(4.5*DTR)*COS((CMOON1-CNODE1)*DTR)*829.91833#
1180 A=(MDM-SDM)^2+DBET^2
1190 B=2*APLAT*DBET
1200 DEL1=(SDIA+MDIA)/2:DEL2=(SDIA-MDIA)/2
```

```
1210 C1=APLAT^2-DEL1^2
1220 C2=APLAT^2-DEL2^2
1230 DISCR1=B^2-4*A*C1
1240 DISCR2=B^2-4*A*C2
1250 CLS
1260 REM *** FIRST-HALF DURATION OF THE ECLIPSE ***
1270 T1=60*ABS((-B-SQR(DISCR1))/(2*A))
1280 PRINT "FIRST-HALF DURATION OF ECLIPSE: ";:N=T1:GOSUB 2200
1290 H=T1*2/5:GOSUB 1120
1300 REM *** SECOND-HALF DURATION OF THE ECLIPSE ***
1310 T4=60*(-B+SQR(DISCR1))/(2*A)
1320 PRINT
1330 PRINT "SECOND-HALF DURATION OF ECLIPSE: ";:N=T4:GOSUB 2200
1340 H=T4*2/5:GOSUB 1120
1350 IF ZZ=1 THEN GOTO 1460
1360 REM *** FIRST-HALF DURATION OF TOTALITY ***
1370 T2=60*ABS((-B-SQR(DISCR2))/(2*A))
1380 PRINT
1390 PRINT "FIRST-HALF OF TOTALITY: ";:N=T2:GOSUB 2200
1400 H=T2*2/5:GOSUB 1120
1410 REM *** SECOND-HALF DURATION OF TOTALITY ***
1420 T3=60*(-B+SQR(DISCR2))/(2*A)
1430 PRINT
1440 PRINT "SECOND-HALF OF TOTALITY: ";:N=T3:GOSUB 2200
1450 H=T3*2/5:GOSUB 1120
1460 PRINT
1470 PRINT TAB(5) "***********************************************"
1480 PRINT TAB(5) "**   SUMMARY OF THE SOLAR ECLIPSE    **"
1490 PRINT TAB(5) "***********************************************"
1500 PRINT TAB(25); " ON";D1; "/";MM; "/";Y; " AT";PLACE$
1510 PRINT TAB(35) "LOCAL MEAN TIME"
1520 PRINT TAB(5) "BEGINNING OF THE ECLIPSE: ";:N=5*H(I)/2-T1
     :GOSUB 2200
1530 H=N*2/5:GOSUB 1120
1540 IF ZZ=1 THEN GOTO 1570
1550 PRINT TAB(5) "BEGINNING OF TOTALITY: ";:N=5*H(I)/2-T2 :GOSUB 2200
1560 H=N*2/5:GOSUB 1120
1570 PRINT TAB(5) "MIDDLE OF THE ECLIPSE: ";:N=5*H(I)/2:GOSUB 2200
1580 H=H(I):GOSUB 1120
```

```
1590 IF ZZ=1 THEN GOTO 1620
1600 PRINT TAB(5) "END OF TOTALITY: ";:N=5*H(I)/2+T3:GOSUB 2200
1610 H=N*2/5:GOSUB 1120
1620 PRINT TAB(5) "END OF THE ECLIPSE: ";:N=5*H(I)/2+T4:GOSUB 2200
1630 H=N*2/5:GOSUB 1120
1640 PRINT TAB(5) "******************************************************"
1645 LOCATE,24:INPUT "DO YOU WANT ANOTHER TRIAL (Y/N)";Y$
1650 IF Y$= "Y" OR Y$= "y" THEN CHAIN "SSRAMOON" ELSE END
1660 REM *** ORIENT ECLIPTIC POINT (SAYANA LAGNA) ***
1670 REM * T: TIME IN IST FOR WHICH LAGNA & MC ARE REQUIRED *
1675 PRINT "T=";T
1680 G1 =6.63627+6.570982*.01*(JJ-2443144!)
1690 TS=G1-INT(G1/24)*24
1700 IF LD>= THEN L=LD+(LM/60)
1710 IF LD<0 THEN L=LD-(LM/60)
1720 S=L/15+TS+(T-5.5)/1436*4
1730 IF S>24 THEN S=S-24
1740 IF S<0 THEN S=S+24
1750 ST=S
1760 T=T-5.5
1770 H=ST+T:IF PD<0 THEN H=H+12
1780 IF H>24 THEN H=H-24
1790 IF H<0 THEN H=H+24
1800 GOSUB 1810:GOTO 1830
1810 MIN=(H-INT(H))*60:TTY=INT((MIN-INT(MIN))*60)
1820 RETURN
1830 IF PD+PM/60>0 THEN PHI=PD+PM/60
1840 IF PD+PM/60<0 THEN PHI=PD-PM/60
1850 S=H:S=S*15
1860 A=S+90:A=A*DTR:W=23.45*DTR
1870 GOSUB 1970
1880 S=H:S=S*15
1890 B=ATN(TAN(A)*COS(W))
1900 T=ATN(COS(A)*TAN(W))
1910 E1=ATN(SIN(A)*SIN(W)*TAN(ABS(PHI*DTR+T)))
1920 L=(B+E1)*RTD
1930 IF PD<0 THEN L=180+L
1940 EF L<0 THEN L=L+360
```

```
1950 IF S<180 THEN L=L+180
1960 TLAG=L
1970 IF PD<0 THEN S=S-180:IF S<0 THEN S=S+360
1980 TRS=S*DTR
1990 IF S=90 THEN MC=90:GOTO 2030
2000 IF S=180 THEN MC=180:GOTO 2030
2010 IF S=270 THEN MC=270:GOTO 2030
2020 A2=RTD*ATN(TAN(TRS)/COS(W))
2030 IF S>90 AND S<180 THEN MC=180+A2
2040 IF S>180 AND S<270 THEN MC=180+A2
2050 IF S<90 OR S>270 THEN M=A2
2060 IF MC<0 THEN MC=MC+360
2070 RETURN
2080 M=ABS(M)
2090 MIN=INT(M):SEC=INT((M-MIN)*60+.5)
2100 IF SEC=60 THEN SEC=0:MIN=MIN+1
2110 PRINT TAB(35);MIN; " ";SEC; " "
2120 RETURN
2130 IF L<0 THEN L=ABS(L)
2150 DEG=INT(L):MIN=(L-DEG)*60:SEC=INT((MIN-INT(MIN))*60+.5)
2160 IF DEG>=360 THEN DEG=DEG-360
2170 IF SEC=60 THEN SEC=0:MIN=MIN+1:IF MIN=60 THEN MIN=0
     :DEG=DEG+1 :IF DEG=360 THEN DEG=DEG-360
2180 PRINT TAB(35);DEG; "°";INT(MIN); "'";SEC; "" "
2190 RETURN
2200 N=ABS(N)
2210 NADI=INT(N):VIN=INT((N-NADI)*60+.5) :IF VIN=60 THEN
     VIN=0:NADI= NADI+1
2220 PRINT TAB(35);NADI;TAB(39); "na.";TAB(43);VIN; "vin.";
2230 RETURN
```

```
10  CLS:REM ** PROGRAM "SSPLA" **
20  PRINT TAB(23); "***********************************************"
30  PRINT TAB(23); "*   PLANETS' POSITIONS ACCORDING TO    *"
40  PRINT TAB(23); "*              SURYA SIDDHANTA            *"
50  PRINT TAB(23); "***********************************************"
60  PI=3.141592653589793#
70  PRINT "——————————————————————"
80  PRINT "**********KUJA SPHLJTA**********"
90  REM ** NO. OF REVNS. 2 296 832 REVOLNS IN 1 577 917 828 CIVIL DAYS **
100 DAILY= 1.455609385509776D-03:REM ** REVOLN
110 MKUJA=KALI*DAILY:REM ** REVOLN **
120 REV=INT(MKUJA):PRINT TAB(15) "REVOLNS SINCE KALI EPOCH: ";REV
130 MKUJA=360*(MKUJA-REV)
140 PRINT "MEAN KUJA AT UJJAYINI MIDNIGHT: ";
150 L=MKUJA:GOSUB 1380
160 REM *** DAILY MOTION: 31' 26" = 0.523888888 DEG ***
170 DAILY=360*DAILY:GOSUB 1290
180 KAALA=(GH+VIG/60)*DAILY/60
190 PRINT "MOTION FOR";GH; " GH";VIG; " VIG: ";
200 L=KAALA:GOSUB 1380
210 PRINT "MEAN KUJA AT GIVEN TIME AT";PLACE$;
220 MKUJA=MKUJA+DESH+KAALA:L=MKUJA:GOSUB 1380
230 K3=235/(2*PI):K4=3/(2*PI):MPLANET=MKUJA:SHIGHROCCA=MRAVI
240 PMA=(129.96/360)+(KALI*204/(CIVIL*1000))
    :REM ** KUJA'S MANDOCCA **
250 PMA = 360*(PMA-INT(PMA))
260 PMA1 =PMA-MKUJA: IF PMA1<0 THEN PMA1=PMA1+360
270 P$= "KUJA": K1=75/(2 * PI): K2=3/(2*PI)
280 GOSUB 1600
290 LOCATE 23,60:PRINT "PRESS ANY KEY TWICE"
300 A$=INPUT$(2)
310 PRINT "**********BUDHA SPHUTA**********"
320 REM ** 17 937 060 REVNS IN 1 577 917 828 CIVIL DAYS **
330 DAILY= 1.136755012314874D-02:REM * REVN *
340 MBUDHA=KALI*DAJILY:REM * REVN *
350 REV=INT(MBUDHA) :PRINT TAB(15) "REVOLNS SINCE KALI EPOCH:
    ";REV
360 MBUDHA=360*(MBUDHA-REV)
370 PRINT "BUDHA-SHIGHRA AT UJJAYINI MIDNIGHT: ";
```

```
380 L=MBUDHA:GOSUB 1380
390 REM *** BUDHA'S DAILY MOTION: 245' 32" = 4.092222222 DEG ***
400 DAILY=360*DAILY:GOSUB 1290:REM * DEG *
410 KAALA=(GH+VIG/60)*DAILY/60
420 PRINT "MOTION FOR";GH; " GH";VIG; " VIG: ";
430 L=KAALA:GOSUB 1380
440 PRINT "SHIGHROCCA AT GIVEN TIME AT";PLACE$;
450 MBUDHA=MBUDHA+DESH+KAALA:L=MBUDHA :GOSUB 1380
460 PRINT "MEAN BUDHA (i.e.RAVI): ";:L=MRAVI :GOSUB 1380
470 K3=133/(2*PI):K4=1/(2*PI)
480 SHIGHROCCA=MBUDHA:MPLANET=MRAVI
490 PMA=(220.32/360)+(KALI*368/(CIVIL*1000))
    :REM ** BUDHA'S MANDOCCA **
500 PMA = 360*(PMA-INT(PMA))
510 PMA1=PMA-MPLANET :IF PMA1<0 THEN PMA1=PMA1+360
520 P$= "BUDHA":K1=30/(2*PI):K2=2/(2*PI)
530 GOSUB 1600
540 LOCATE 23,60:PRINT "PRESS ANY KEY TWICE"
550 A$=INPUT$(2)
560 PRINT "------------------------------------"
570 PRINT "**********GURU SPHUTA**********"
580 REM ** 364 220 REVOLNS IN 1 577 917 828 CIVIL DAYS **
590 DAILY=2.3082317313167460-04:REM * REVN *
600 MGURU=KALI*DAILY:REM * REVNS *
610 REV=INT(MGURU) :PRINT TAB(15) "REVOLNS SINCE KALI EPOCH: ";REV
620 MGURU=360*(MGURU-REV)
630 PRINT "MEAN GURU AT UJJAYINI MIDNIGHT: ";
640 L=MGURU:GOSUB 1380
650 REM *** GURU'S DAILY MOTION: 4' 59" = 0.083055555 DEG ***
660 DAILY=360*DAILY:GOSUB 1290
670 KAALA=(GH+VIG/60)*DAILY/60
680 PRINT "MOTION FOR ";GH; " GH";VIG; " VIG: ";
690 L=KAALA:GOSUB 1380
700 PRINT "MEAN GURU AT GIVEN TIME AT";PLACE$;
710 MGURU=MGURU+DESH+KAALA:L=MGURU:GOSUB 1380
720 K3=70/(2*PI):K4=-2/(2*PI):MPLANET=MGURU:SHIGHROCCA=MRAVI
730 PMA=(171/360)+(KALI*900/(CIVIL*1000)) :REM ** GURU'S MANDOCCA
740 PMA = 360*(PMA-INT(PMA))
750 PMA1=PMA-MGURU:IF PMA1<0 THEN PMA1=PMA1+360
```

```
760  P$= "GURU":K1=33/(2*PI):K2=1/(2*PI)
770  GOSUB 1600
780  LOCATE 23,60:PRINT "PRESS ANY KEY TWICE"
790  A$=INPUT$(2)
800  PRINT "————————————————————————————"
810  PRINT "**********SHUKRA SPHUTA**********"
820  REM ** 7 022 376 REVNS IN 1 577 917 828 CIVIL DAYS **
830  DAILY= 4.450406653241768D-03:REM * REVN *
840  MSHUKRA=KALI*DAILY: REM * REVNS *
850  REV=INT(MSHUKRA)
     :PRINT TAB(15) "REVOLNS SINCE KALI EPOCH: ";REV
860  MSHUKRA=360*(MSHUKRA-REV)
870  PRINT "SHUKRA-SHIGHRA AT UJJAYINI MIDNIGHT: ";
880  L=MSHUKRA:GOSUB 1380
890  REM *** SHUKRA'S DAILY MOTION: 96' 7" 43' " 37.3" "
900  DAILY=360*DAILY:GOSUB 1290:REM * DEG *
910  KAALA=(GH+VIG/60)*DAILY/60
920  PRINT "MOTION FOR";GH; " GH";VIG; " VIG: ";
930  L=KAALA:GOSUB 1380
940  PRINT "SHIGHROCCA AT GIVEN TIME AT";PLACE$;
950  MSHUKRA=MSHUKRA+DESH+KAALA:L=MSHUKRA:GOSUB 1380
960  PRINT "MEAN SHUKRA (i.e.RAVI): ";:L=MRAVI:GOSUB 1380
970  K3=262/(2*PI):K4=2/(2*PI)
980  SHIGHROCCA=MSHUKRA:MPLANET=MRAVI
990  PMA=(79.65/360)+(KALI*535/(CIVIL*1000))
     :REM ** SHUKRA'S MANDOCCA **
1000 PMA=360*(PMA-INT(PMA))
1010 PMA1=PMA-MPLANET: IF PMA1<0 THEN PMA1=PMA1+360
1020 P$= "SHUKRA": K1=12/(2*PI):K2=1/(2*PI)
1030 GOSUB 1600
1040 LOCATE 23,60:PRINT "PRESS ANY KEY TWICE"
1050 A$=INPUT$(2)
1060 PRINT "**********SHANI SPHUTA**********"
1070 REM ** 146 568 REVNS IN 1577197828 CIVIL DAYS **
1080 DAILY=9.28869662280031D-05 :REM * REVN *
1090 MSHANI=KALI*DAILY:REM * REVNS *
1100 REV=INT(MSHANI) :PRINT TAB(15) "REVOLNS SINCE KALI EPOCH: ";REV
1110 MSHANI=360*(MSHANI-REV)
1120 PRINT "MEAN SHANI AT UJJAYINI MIDNIGHT: ";
```

```
1130 L=MSHANI:GOSUB 1380
1140 REM ** SHANI'S DAILY MOTION: 2' 0" 22' " 53.4" " **
1150 DAILY=360*DAILY:GOSUB 1290:REM * DEG *
1160 KAALA=(GH+VIG/60)*DAILY/60
1170 PRINT "MOTION FOR";GH; " GH";VIG; " VIG: ";
1180 L=KAALA:GOSUB 1380
1190 PRINT "MEAN SHANI AT GIVEN TIME AT";PLACE$;
1200 MSHANI=MSHANI+DESH+KAALA:L=MSHANI:GOSUB 1380
1210 K3=40/(2*PI):K4=1/(2*PI):MPLANET=MSHANI:SHIGHROCCA=MRAVI
1220 PMA=(236.61/360)+(KALI*39/(CIVIL* 1000))
     :REM ** SHANI'S MANDOCCA **
1230 PMA=360*(PMA-INT(PMA))
1240 PMA1=PMA-MSHANI:IF PMA1<0 THEN PMA1=PMA1+360
1250 P$= "SHANI":K1=49/(2*PI):K2=1/(2*PI)
1260 GOSUB 1600
1270 LOCATE 23,60:PRINT " END OF THE PROGRAM"
1280 END
1290 REM *** DESHANTARA CORRECTION ***
1300 DESH=ULAM*DAILY/360
1310 PRINT "DESHANTARA CORRECTION: ";
1320 IF DAILY<0 AND ULAM>0 THEN PRINT TAB(39) "-";:GOTO 1360
1330 IF DAILY<0 AND ULAM<0 THEN PRINT TAB(39) "+";:GOTO 1360
1340 IF ULAM>0 THEN PRINT TAB(39) "+";
1350 IF ULAM<0 THEN PRINT TAB(39) "-";
1360 L=DESH:GOSUB 1380
1370 RETURN
1380 IF L<0 THEN L=ABS(L)
1390 DEG=INT(L):MIN=(L-DEG)*60:SEC=INT((MIN-INT(MIN))*60+.5)
1400 IF DEG>=360 THEN DEG=DEG-360
1410 IF SEC=60 THEN SEC=0:MIN=MIN+1:IF MIN=60 THEN MIN=0
     :DEG=DEG+1:IF DEG>=360 THEN DEG=DEG-360
1420 PRINT TAB(40);DEG; "°";INT(MIN); "'";SEC;"""
1430 RETURN
1440 REM *** SHIGHRA EQNS. ***
1450 IF P$<> "BUDHA" AND P$<> "SHUKRA" AND Z=0
     THEN PRINT "SHIGHROCCA: ";:L=SIRGHROCCA:GOSUB 1380
1460 PRINT "SHIGHRA ANOMALY: ";
1470 L=PMK:GOSUB 1380
1480 PI= 3.14159256#
```

```
1490 PMK=PMK*PI/180:REM * RADIAN *
1500 SN=SIN(PMK):CS=COS(PMK)
1510 K=(PI/180)*(K3-K4*ABS(SN))
     :REM ** COR.SHIGHRA RADIUS IN RADIANS **
1520 DPL=K*SN:REM ** DOHPHALA **
1530 SKR=SQR(K*K+2*K*CS+1):REM ** SIGHRAKARNA **
1540 SE=(180/PI)*ATN(ABS(DPL/SQR(SKR*SKR-DPL*DPL)))
     :REM ** SHIGHRA EQN **
1550 IF PMK>PI THEN SE=-SE
1560 PRINT "SHIGHRA EQN: ";:IF PMK<PI THEN PRINT TAB(39) "+";
1570 IF PMK>PI THEN PRINT TAB(39) "-";
1580 L=SE:GOSUB 1380
1590 RETURN
1600 REM *** MANDA and SHIGHRA CORRECTIONS ***
1610 PMK=SHIGHROCCA-MPLANET:IF PMK<0 THEN PMK=PMK+360
     :REM ** SHIGHRA ANOMALY**
1620 Z=0:GOSUB 1440
1630 P1=MPLANET+SE/2 :REM ** PLANET'S LONG. AFTER 1st OPERATION **
1640 PRINT "LONG.after I st COR.( SE/2)";
1650 L=P1:GOSUB 1380
1660 PMA1=PMA-P1
     : REM ** PLANET'S MANDA ANOMALY AFTER 1st OPERN. **
1670 IF PMA1<0 THEN PMA1 =PMA1+360
1680 GOSUB 1900
1690 PRINT "LONG.after 2nd COR.(ME/2): ";
1700 P2 =-P1+PEQ/2: REM ** PLANET'S LONG. AFTER 2nd OPERATION **
1710 L=P2:GOSUB 1380
1720 PMA2=PMA-P2: IF PMA2<0 THEN PMA2=PMA2+360
1730 PRINT " COR.";:Z=Z+1
1740 PMA1=PMA2:GOSUB 1900:REM ** PLANET'S EQN.OF CENTRE **
1750 PRINT "LONG.after 3rd COR.(ME): ";
1760 P3=MPLANET+PEQ:IF P3>360 THEN P3=P3-360
1770 IF P3<0 THEN P3=P3+360
1780 L=P3:GOSUB 1380
1790 PMK = SHIGHROCCA - P3:IF PMK<0 THEN PMK=PMK+360
1800 PRINT "COR.";
1810 GOSUB 1440
1820 PRINT "LONG.after 4th COR.(SE): ";
1830 PTL=P3+SE
1840 L=PTL:GOSUB 1380
```

```
1850 PRINT "————————————————————————"
1860 PRINT P$; "'S TRUE LONGITUDE: ";
1870 L=PTL:GOSUB 1380
1880 PRINT "————————————————————————"
1890 RETURN
1900 REM **** EQUATION OF CENTRE ****
1910 IF Z=0 THEN PRINT "MANDOCCA: ";:L=PMA:GOSUB 1380
1920 PRINT "MANDA ANOMALY: ";
1930 L=PMA1:GOSUB 1380
1940 PRINT "MANDA EQUATION: ";
1950 PMA1=PMA1 *PI/180:REM **RADIANS**
1960 SN=SIN(PMA1)
1970 PEQ=(K1-K2*ABS(SN))*SN:REM ** PLANET'S EQN.OF CENTRE **
1980 IF PMA1<PI THEN PRINT TAB (39) "+";
1990 IF PMA1>PI THEN PRINT TAB(39) "-";
2000 L=PEQ:GOSUB 1380
2010 RETURN
```

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Glossary of Technical Terms in Indian Astronomy

I. English to Sanskrit

Altitude	*Unnata, Unnati*
Anomaly	*Manda Kendra* or *Śīghra Kendra*
Apogee or aphelion	*Mandocca*
Apse line	*Nīcoccarekhā*
Ascending node of the Moon	*Rāhu*
Celestial equator	*Viṣuvadvṛtta, Viṣuvadvalaya, Nāḍivṛtta*
Celestial latitude	*Kṣepa, Vikṣepa, Śara*
Celestial meridian	*Yāmyottara maṇḍala*
Co-azimuth	*Digamśa*
Co-latitude	*Lamba*
Declination	*Apakrama, Krānti, Apama*
Ecliptic	*Apama maṇḍala, Krāntivṛtta*
Epicycle	*Nīcoccavṛtta, Anuvṛtta*
Equation of centre	*Mandaphala*
Equation of conjunction	*Śīghraphala*
Equatorial horizon	*Nirakṣa kṣitija*
Equinoctial shadow	*Akṣabhā, palabhā*
Equinox	*Krāntipāta*
Gnomon	*Śaṅku*
Hemisphere	*Kapāla*
Hypotenuse	*Karṇa, Śrava*
Meridian zenith distance	*Avanati*
Orbit	*Kakṣā*

Parallax in latitude	*Nati, Avanati*
Parallax in longitude	*Lambana*
Perigee	*Mandanīca*
Polar longitude	*Dhruvaka*
Pole of the celestial equator	*Dhruva*
Precession of the equinoxes	*Ayanāmśa, Ayanacalana, Krāntipātagati*
Solstice	*Ayanānta*
Zenith distance	*Natāmśa*
Zodiac	*Bhacakra*

II. Sanskrit to English

Abda	Year
Abhijit	Alpha Lyrae
Acala	Successive aproximation
Adhikamāsa	Additional lunar (intercalary) month in a lunar year
Ādyantakāla	Duration of an eclipse
Agastya	Star Canopus
Agni	Star Beta Taurii
Agra	Amplitute, i.e., the arc along the cel. horizon lying between the east point and the rising point of a heavenly body
Agrajyā (*agrajīvā*)	Rsine of amplitude
Ahargaṇa	Number of civil days elapsed on a given day since a chosen epoch; *dinagaṇa, dyugaṇa*
Ahorātra viṣkambha	Diameter of the diurnal circle
Ahorātra vṛtta	Diurnal circle
Aja	*Meṣa* (Aries sign)
Akṣa (*akṣāmśa*)	Terrestrial latitude
Akṣabhā (*palabhā*)	Equinoctial shadow
Akṣakoṭi	Co-latitude; Rsine of co-latitude or Rcosine of latitude
Akṣavalana (*valanāmśa*)	Angle subtended at a heavenly body on the ecliptic by the arc joining the north pole of the cel. equator and the north point of the cel. horizon
Amāvāsyā	Last day of a lunar month, new moon
Amśa (*amśaka*)	Degree, fraction

Aṅgula	Approximately one inch; $^1/_{12}$ of the length of the standard gnomon (*śaṅku*)
Antyaphala	Maximum equation of centre (*mandaphala*) or of conjunction (*śīghraphala*)
Anuloma	Direct motion of a planet (opp. of *Vakra*)
Anurādhā	Delta Scorpionis
Apakrama	Declination
Apakrama maṇḍala	Ecliptic
Apama	Declination
Apara	West (*paścima*)
Ardhakarṇa	Semi-diameter (i.e., radius)
Ardharātra	Midnight
Ārdharātrika	Calculations from the midnight
Āsanna	Approximate; *asphuṭa*
Asitapakṣa	Dark half of the lunar month (*Kṛṣṇapakṣa*)
Asphuṭa	Approximate; *āsanna, anityam*
Asu	$^1/_6$th of a *Vighaṭī* (i.e., 4 seconds)
Avanati	Parallax in cel. latitude (*nati*)
Ayanacalana	Precession of the equinoxes
Ayanāṁśa	Amount of precession of equinoxes (in degrees)
Ayanānta	Solstice
Bāhu	Base of a right-angled triangle for an angle θ, its *bhuja* is θ, $180°-\theta$, $\theta-180°$, $360°-\theta$ according as θ is in I, II, III, IV quadrant; *bhuja*
Bhacakra	Zodiac (consisting of 27 *nakṣatras* or 12 *rāśis*)
Bhagaṇa	Revolutions of a celestial body in a long period of time (like *mahāyuga*); *paryāya*
Bhāga	One degree of arc; *amśa*
Bhagaṇa kāla	Sidereal period of a heavenly body
Bhagola	Celestial (starry) sphere
Bhoga	Portion
Bhogya khaṇḍa	Tabular difference of Rsine, etc. yet to be covered
Bhogya mandaphala	Portion of equation of centre yet to be covered
Bhūcchāyā	Earth's shadow
Bhūgola	Terrestrial globe
Bhuja	See *bāhu*
Bhujajyā	Rsine of *bhuja*; *dorjyā*

Bhujāntara	Correction for the equation of time due to the eccentricity of the orbit
Bhūkarṇa	Diameter of the Earth; *bhūvyāsa*
Bhukta	Covered (or traversed) already
Bhuktagati (*-phalāmśa*)	Increase in the last *śīghrakendra* covered in order to find the true *śīghraphala*
Bhukti	Daily motion of a heavenly body; *dinagati*
Bīja	Correction (to the parameters for cel. longitude, cel. latitude, etc.)
Bimba	Disc or diameter of a body (particularly of the Moon, the Sun or the Earth's shadow-cone in an eclipse)
Bimbārdha	Semi-diameter (i.e., radius) of the disc of a heavenly body
Cakra	Circle
Cakraliptā	Minutes of arc in a circle; $360 \times 60 = 21{,}600'$
Cakrāmśa	Degrees in a circle, 360°
Cala	Variable
Candra	Moon
Candrakarṇa	Distance of the Moon from the Earth's centre
Candramāsa	Lunar month; *cāndramāsa*
Candranatajyā	Rsine of the zenith distance of the Moon
Candra (vyāsa) bimba	Angular diameter of the Moon
Cara	Arc on the cel. equator between 6'o clock circle and the hour circle of a heavenly body at rising
Cara samskāra	Correction to the mean position of a heavenly body due to the difference between instants of midnight at Laṅkā and the given place
Caturyuga	A great age of 43,20,000 years; *mahāyuga*
Chādaka	Eclipser; *Grāhaka*
Chādya	Eclipsed body; *Grāhya*
Chandrakalā	$^{1}/_{16}{}^{th}$ of the Moon's disc
Chāya	Shadow
Chāyākarṇa	Hypotenuse of the right-angled triangle whose other two sides are the gnomon (*śaṅku*) and its shadow
Cheda	Division; denominator
Dakṣiṇa	South, *Yāmya*
Dakṣiṇāyana	Southern course of the Sun
Darśa	Conjunction of the Sun and the Moon

Deśa	Place
Deśāntara	Difference in longitudes of a given place and the prime meridian (usually Ujjayinī)
Dhana	Positive; excess
Dhanurbhāga	Arc of 225′ ($3\,^3/_4^\circ$)
Dhruva	Pole star; fixed
Dhruvaka	Polar longitude along the ecliptic at the foot of the circle from the pole
Digamśa	Co-azimuth
Dina	Day
Dinagaṇa	Civil days; *Ahargaṇa*
Dinārdha	Half-day
Diśā	Direction
Dorjyā	Rsine of the base of a right angled triangle; *bhujajyā*
Dṛgguṇa	Rsine of the zenith distance
Dṛglambana	Parallax in zenith distance
Dvāparayuga	Third of the four *yugas* of a *mahāyuga*; duration: 8,64,000 years
Ekāyanagata	Bodies having the same declination
Gati	Motion; usually daily motion
Ghana	Cube
Ghāta	Multiplication, product
Ghaṭikā	A unit of time; $^1/_{60}$th part of a day (24 minutes); *nāḍī, nāḍikā; ghaṭī*
Gola	Globe, sphere
Graha	Planet; a moving heavenly body
Grāhaka	Eclipsing body, eclipser; *chādaka*
Grahaṇa	Eclipse; *uparāga, upalapti*
Grahaṇa madhya	Middle of eclipse
Graha(yoga)yuti	Conjunction of planets
Grāhya	Eclipsed body; *chādya*
Grāsa	Measure of obscuration (eclipse)
Grāsamāna	Magnitude of eclipse
Guṇa	Multiple
Harija	Parallax in longitude; *lambana*
Horā	One hour; $^1/_{24}$th of a day

Indūcca	Moon's apogee; *mandocca* of the Moon
Iṣṭa	Desired or given
Iṣu	Rversine i.e., R – Rcosine of an angle where R is the radius of the deferant circle (R = 3438′ according to the **Ārya-bhaṭīyam**)
Kadamba	Pole of the ecliptic
Kadambaprota	Secondary to the ecliptic
Kakṣā	Orbit (or path) of a heavenly body
Kalā	A unit of measurement of angles; $^{1}/_{60}{}^{\text{th}}$ of one degree, i.e., 1 minute of arc; *liptā, liptikā*
Kāla	Time
Kaliyuga	Last of the *yugas* of a *mahāyuga*; duration: 4,32,000 years
Kapāla	Hemisphere
Karaṇa	Half of a lunar day (*tithi*)
Karṇa	Diameter; hypotenuse of a right-angled triangle; *śrava*
Kendra	Anomaly of a heavenly body, angular distance from its *mandocca* (apogee) or *śīghrocca* (apex point of conjunction)
Ketu	Descending node of the Moon; 180° – *Rāhu*
Khagola	Celestial sphere
Khamadhya	Zenith
Koṇa	Angle
Koṭi	Perpendicular
Koṭijyā	Rcosine of an angle
Krānti	Declination; *apakrama*
Krāntipāta	Equinoxes, points of intersection of the ecliptic and the cel. equator; *viṣuva, viṣuvat*
Kṛṣṇapakṣa	Dark half of a lunar month
Kṛtayuga	First of the four *yugas* consituting a *mahāyuga*; duration: 17,28,000 years
Kṣaṇa	A unit of time; 2 *ghaṭīs* (according to **Sūryasiddhānta**)
Kṣepa	An additive quantity; cel. latitude; *Vikṣepa, śara*
Kṣitija	Horizon
Lagna	Ascendant; orient ecliptic point; point of intersection of the ecliptic with the eastern horizon
Lamba	Co-latitude of a place
Lambāṃśa	Co-latitude of a place in degrees
Lambana	Parallax in longitude

Laṅkā	Place on the Earth's equator through which the prime meridian (through Ujjayinī) passes
Liptā (*Liptikā*)	Minute of arc; *kalā*
Madhyabhā	Meridian ecliptic point
Madhya bhukti	Mean daily motion
Madhyacchāyā	Midday shadow of the gnomon
Madhya dina	Mid-day
Madhya gati	Mean (daily) motion
Madhya graha	Mean longitude of a planet
Madhya grahaṇa	Middle of an eclipse
Madhyāhna	Mid-day
Madhyāhna cchāyā	Mid-day shadow
Madhyāhna paridhi	Circumference of the epicycle at mid-day
Madhyajyā	Rsine of the zenith distance of the meridian ecliptic point
Madhyakarṇa	Radius; *vyāsārdha, karṇārdha*
Madhya krānti	Declination of the meridian ecliptic point
Madhya lagna	Meridian ecliptic point
Madhya lambana	Parallax in longitude at the middle of eclipse
Madhyama krānti	Declination obtained from mean longitude
Madhya rātra	Midnight
Madhya rekhā	Central meridian of the earth passing through Laṅkā, Ujjayinī, Kurukṣetra and Mount Meru; *Madhyasūtra*
Madhya śaṅku	Gnomon formed with the Sun crossing the meridian
Madhya sthityardha	Intervals between the middle of an eclipse and the moments of contact and separation (end)
Mahāśaṅku	Rsine of altitude
Mahāyuga	*Caturyuga:* A great age consisting of 432×10^4 years; consists of four parts (*yugas*) viz., *Kṛta, Tretā, Dvāpara* and *Kali* of durations in the ratio 4:3:2:1. According to the **Āryabhaṭīyam**, the four *yugas* are of equal durations: 108×10^4 years
Manda	Slow
Maṇḍala	Circle; revolution; *vṛtta, cakra*
Mandakarman	The process of finding the equation of centre
Mandakendra	Angular distance of a heavenly body from its *mandocca*; anomaly from apogee or aphelion
Mandanīcocca vṛtta	Manda epicycle; *mandānuvṛtta*

Mandaphala	Equation of centre (due to the eccentricity of an orbit)
Mandanīca	Perigee; 180° from *mandocca*
Mandocca	Apex of the slowest motion; apogee or aphelion
Manvantara	Age of Manu: 3,08,448,000 solar years
Māsa	Month
Melaka	Conjunction of planets; *yuti, yoga*
Meru	Earth's pole
Meṣa caradala	Ascensional difference at the end of *Meṣa rāśi* (the first sign of the zodiac)
Mithuna caradala	Ascensional difference at the end of *Mithuna rāśi*
Mokṣa	End (i.e., the last contact) of an eclipse
Mokṣa lambana	Parallax in longitude at the end of an eclipse
Mṛgavyādha	Alpha Canis Major; Sirius; *Lubdhaka*; brightest visible star in the sky
Muhūrta	A unit of time; 30 *muhūrtas* = 1 day; 1 *muhūrta* = 2 *ghaṭikās* = 48 minutes
Mūla	Square-root
Nākṣatra dina	Sidereal day
Nata	Hour angle; zenith distance
Natakālajyā (*Natajyā*)	Rsine of the hour angle
Natakāla koṭijyā	Rcosine of the hour angle
Nati	Parallax in cel. latitude; *avanati*
Nāḍikā (*Nāḍī*)	$^{1}/_{60}{}^{th}$ of a day; 24 minutes; *ghaṭikā*
Nemi	Circumference of a circle; *paridhi*
Nīcoccarekhā	Line of apse
Nīcoccavṛtta	Epicycle; *anuvṛtta*
Nimeṣa	A unit of time; according to the **Siddhāntaśiromaṇi**, 1 day = 9,72,000 *nimeṣas*
Nimīlana	Total obscuration of eclipsed body
Nirakṣa	Earth's equator; zero latitude
Nirakṣa kṣitija	Equatorial horizon
Niśā	Night
Oja	Odd; usually w.r.t. 1^{st} and 3^{rd} quadrants
Pāda	A quarter; quadrant
Pakṣa	Fortnight; half of a lunar month
Pala	A unit of time; *vināḍī*; 24 seconds

Palabhā	Equinoctial shadow; *akṣabhā*
Parama Krānti	Maximum declination
Parama lambana	Horizontal parallax; maximum parallax
Pāramārthika valana	Algebraic sum of *akṣavalana* and *ayana valana* (numerical addition when both are north or both south and difference when they are of opposite directions)
Paridhi	Circumference; *nemi*
Parilekhā	Projection
Parvamadhya	Middle of an eclipse
Parvanāḍī	Instant of conjunction or opposition of the Sun and the Moon in *nāḍīs*
Parvānta	End of a *parva* (lunar fortnight)
Paryāya	Revolutions of a heavenly body in a given period (like *mahāyuga*); *bhagaṇa*
Paścima	West
Pāta	Node of the Moon (or a planet); point of intersection of the ecliptic with the orbit (*kakṣā*) of the Moon (or a planet)
Phala	Result (like *mandaphala* and *śīghraphala*)
Pragrāsa	Beginning of an eclipse; first contact of an eclipse
Prāhṇa	Forenoon; *pūrvāhna*
Prakṣepa	Addition
Prāṇa	A unit of time, 4 seconds of sideral time; *asu*
Pratiloma	Retrograde; *vakra*
Pratipada	The first *tithi* (lunar day) of either bright or dark half of a lunar month
Pūrṇimā	Full moon; opposition of the Sun and the Moon
Pūrṇimānta	End of the full-moon day
Pūrva	East
Pūrvāhṇa	Forenoon
Rāhu	Ascending node of the Moon; *pāta* of the Moon; *tamas*
Rāhumāna	Angular diameter of the Earth's shadow
Rāśi	Zodiacal constellation; each of 30° extent
Rāśi kalā	Number of minutes of arc in a zodiacal sign; $30 \times 60 = 1800'$
Ravikarṇa	Distance of the Sun from the Earth's centre
Ravi paramalambana	Horizontal parallax of the Sun
Rekhā	Prime meridian
Ṛju (*gati*)	Direct (motion), as opposed to retrograde
Ṛtu	Season; a year consists of six *ṛtus*

Śaka	Era of King Śālivāhana commencing in the year 78 AD, *Śālivāhana śaka*
Sama	Even
Samamaṇḍala	Prime vertical
Samāsavṛtta	Circle of radius equal to the sum of the radii of the eclipsing and the eclipsed bodies
Saṅkrānti	Solar ingress into a sign of the zodiac
Śaṅku	Gnomon; *kīlaka*
Śara	Celestial latitude; *vikṣepa*
Sauradina	Solar day
Sauramāsa	Solar month
Sauravarṣa	Solar year
Sāvanadina	Civil day; *kudina, bhūdivasa*
Śīghra	Fast motion (as opposed to *manda*)
Śīghrakarman	Finding out the equation of 'conjunction' (*śīghraphala*)
Śīghrakendra	Angular distance of a mean planet from its *śīghrocca* (apex of 'conjunction')
Śīghra nīcocca vṛtta	Epicycle of *śīghra*
Śīghraphala	Equation of conjunction
Śīghrocca	Apex of fast (*śīghra*) motion. In the case of superior planets the mean Sun is the *śīghrocca* while for Mercury and Venus two special points are defined as their *śīghrocca*. However, Nīlakaṇṭha Somayāji maintains that the Sun must be the *śīghrocca* for all the planets
Sparśakāla	Time of first contact of an eclipse
Sparśa lambana	Parallax at the beginning of an eclipse
Spaṣṭagati	True motion of a planet; *sphuṭagati*
Spaṣṭagraha	True planet; *sphuṭagraha*
Sphuṭagati	True motion of a planet; *spaṣṭagati*
Sphuṭagraha	True planet; *spaṣṭagraha*
Sphuṭavikṣepa	Celestial latitude corrected by parallax
Śrava	Hypotenuse; *karṇa*
Śrāviṣṭhā	Beta Delphini; *Dhaniṣṭhā*
Śuklapakṣa	Bright half of a lunar month
Tamas	Ascending node of the Moon; *Rāhu*
Tantra	Indian astronomical texts which adopt the beginning of *Kaliyuga* as the epoch

Tārāgraha	Star-planet, referring to Mars, Mercury, Jupiter, Venus, Saturn
Tāraka (*Tārā*)	Star; asterism
Tiṣya	Delta Cancri; *Puṣya*
Tithi	Lunar day; $^{1}/_{30}{}^{\text{th}}$ of a lunar month
Trairāśika	Rule of three
Tretāyuga	Second of the four parts of a *mahāyuga*; duration: 12,96,000 years
Tribhoṇa lagna	Nonagesimal point; point of the ecliptic at the shortest distance from the zenith; 90° less than *Lagna*
Ucca	Apogee; *mandocca*
Udaya	Rising
Udayajyā	Orient sine; Rsine of the amplitude of *lagna* from the east
Udayāntara samskāra	The correction to the cel. longitude on account of the equation of time due to obliquity of the ecliptic; complementary to the *bhujāntara*
Udayaprāṇa	Duration of the rising of the signs (*rāśis*) in *prāṇa* (or *asu*) unit of time; *udayāsu*
Udvṛtta	Equinoctial circle; east and west hour circle; 6'o clock circle
Unmīlana	End of totality of eclipse
Unnatakāla	Distance from the horizon in time unit; time elapsed after rising of a celestial body
Unnatāmśa	Complement of zenith distance
Unnati	Altitude
Upapluti	Eclipse; *grahaṇa, uparāga*
Ūrdhva yāmyottara vṛtta	Celestial meridian
Uttara	North
Uttaragola	Northern hemiphere; *Uttara Kapāla*
Uttarāyaṇa	Northern course of the Sun along the ecliptic
Vaidhṛta	The "aspect" between the Sun and the Moon when the sum of their longitudes is 360° and their declinations (*krānti*) are equal in magnitude but opposite in directions
Vakragati	Retrogression
Vāra	Week day; *vāsara*
Varṣa	Year; *abda*
Vikala	Remainder; *śeṣa*

Vikalā	One second of arc; $^{1}/_{3600}{}^{th}$ of a degree; *viliptā*
Vikalagati	Stationary motion
Vikrama samvat	Era named after King Vikrama starting from 57–56 BC
Vikṣepa	Polar latitude; cel. latitude; *kṣepa, śara*
Viliptā	One second of arc; *vikalā*
Vilomagati	Retrograde motion; *Vakragati*
Vimarda	Totality of an eclipse
Vināḍī	$^{1}/_{60}{}^{th}$ of a *nāḍī*; 24 seconds; *pala, vighaṭī*
Vipala	$^{1}/_{60}{}^{th}$ of a *pala*
Viṣama	Odd
Viṣkambha	Diameter of a circle; *vyāsa*
Viṣuva (*Viṣuvat*)	Equinox; *Krāntipāta*
Vitasti	A length of 12 *aṅgulas*; approximately one foot
Vitribha lagna	*Lagna* – 90°; *tribhoṇa lagna*; nonagesimal
Vṛṣacaradala	Ascensional difference at the end of *Vṛṣabha* (second sign of zodiac)
Vṛtta	Circle; *maṇḍala, cakra*
Vyakṣa	Terrestrial equator; zero latitude; *nirakṣa*
Vyāsa	Diameter of a circle; *viṣkambha*
Vyāsārdha	Radius of a circle
Vyatipāta	The "aspect" of the Sun and the Moon when the sum of their longitudes is 180° and their declinations (*krānti*) are same both in magnitude and direction
Yāmya	South; *dakṣiṇa*
Yāmyagola	Southern hemisphere
Yoga tārā	Principal star in an asterism
Yoga (*Yuti*)	Conjunction; *melaka*
Yojana	A unit of distance; usually about 5 miles, but taken differently by different authors
Yuga	An age (of long duration)
Yugma	Even; couple

Index

This book is a survey of the development of astronomy in India from the Vedic times to the present. It is intended to create an awareness about Indian Astronomy among interested students and general readers alike. The systematically planned contents and a lucid style enable the reader to be fairly proficient in the concepts, techniques and computational procedures developed by the great Indian astronomers, like Aryabhata, Brahmagupta and Bhaskara-II, over more than a millennium and a half. This is the first comprehensive book of its kind on Indian Astronomy. It has a unique feature in that it provides, for the first time, *Computer Programs* for the computations of true planetary positions, Lunar and Solar Eclipses. A detailed bibliography, glossary and index further enhance the usefulness of this book.

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